Assessment Title: Box Problem
Unit 4: Quadratic Functions: Modeling

Learning Targets:

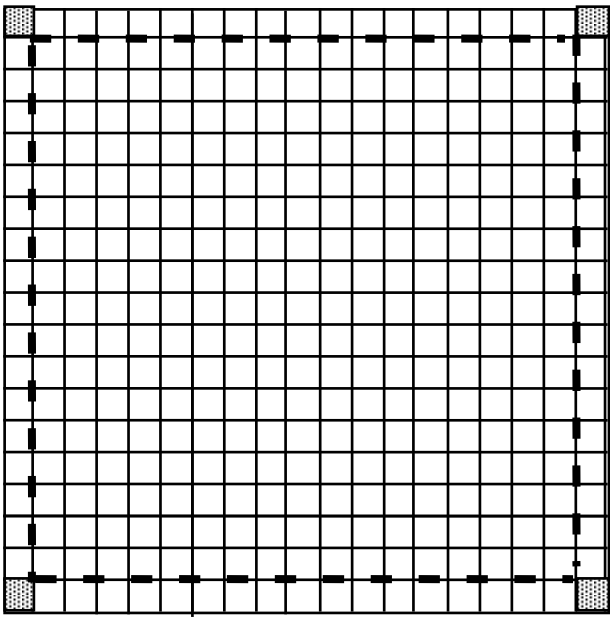
- I can show that the rate of change for a quadratic function is linear.

The Box Problem

You have been given a square sheet of card stock with a 19 by 19 grid on it, like the one shown below.

1. **Flat Box**
   a) Fill in the following entries in the first row of the table for the “flat box” represented by your square sheet of paper:
      a) Height
      b) Perimeter
      c) Area
      d) Surface Area

2. **Box #1**
   a) Tear (or cut) one square out of each corner of the paper so that your paper looks like the diagram to the right.
   b) Fold the side "flaps" up to form an open box.
   c) Fill in the following entries in the second row of the table
      i) Height
      ii) Perimeter
      iii) Area
      iv) Surface Area
      v) Volume



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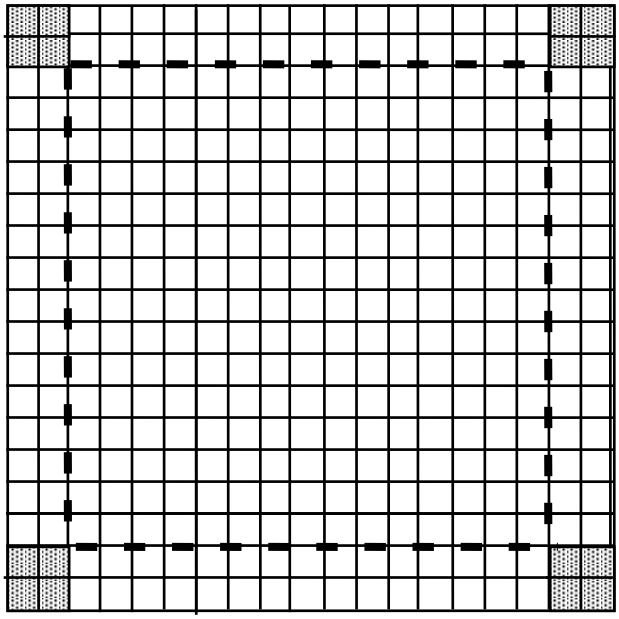
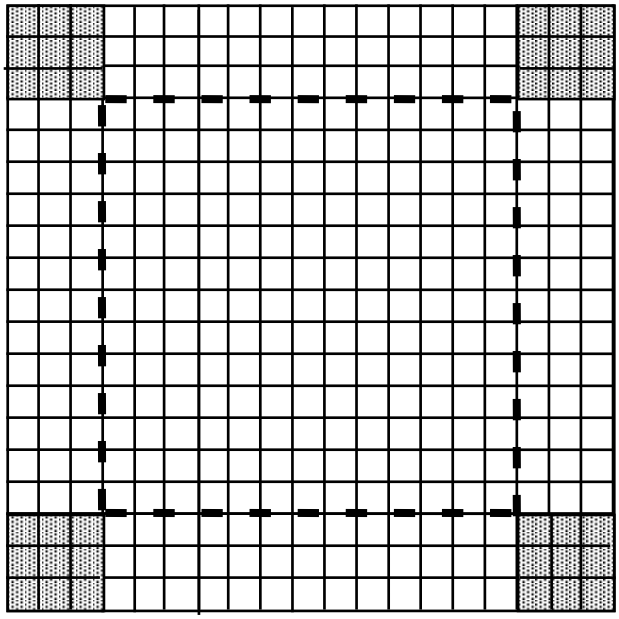
3. **Box #2**
   a) Tear three additional squares out of each corner of the paper so that your paper looks like the diagram to the right.
   b) Fold the side "flaps" up to form an open box.
   c) Fill in the following entries in the third row of the table
      i) Height
      ii) Perimeter
      iii) Area
      iv) Surface Area
      v) Volume



4. **Box #3**
   a) Tear five additional squares out of each corner of the paper so that your paper looks like the diagram to the right.
   b) Fold the side "flaps" up to form an open box.
   c) Fill in the following entries in the third row of the table
      i) Height
      ii) Perimeter
      iii) Area
      iv) Surface Area
      v) Volume



5. **Box #4-10**
   a) Continue to tear out additional squares from the four corners following the pattern you have established and fold your box each time.
   b) After each tear, fill in the following entries in the respective row of the table
      i) Height
      ii) Perimeter
      iii) Area
      iv) Surface Area
      v) Volume

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6. Now fill in the remaining eight columns in the table.

a. In the column labeled ΔP, calculate the ***changes*** in the perimeter.

For example, in the second row for the column labeled ΔP, you should enter the value of (Perimeter of the base for height 1) – (Perimeter of the base for height 0)

b. In the column labeled ΔA, calculate the ***changes*** in the area.

For example, in the second row for the column labeled ΔA, you should enter the value of (Area of the base for height 1) – (Area of the base for height 0)

c. In the column labeled $\Delta^2 A$, calculate the ***changes*** in ΔA (which represents the changes in the changes in A).

For example, in the third row for the column labeled $\Delta^2 A$, you should enter the value of (ΔA from the third row) – (ΔA from the second row)

d. Compute the values for ΔS and $\Delta^2 S$ (the change in the surface area and the change in the change in surface area) in a similar manner to what you did for ΔA and $\Delta^2 A$.

e. Compute the values for ΔV and $\Delta^2 V$ (the change in the volume and the change in the change in the volume) in a similar manner to what you did for ΔA and $\Delta^2 A$.

f. In the column labeled $\Delta^3 V$, calculate the ***changes*** in $\Delta^2 V$ (which represents the changes in the changes inΔV, or the changes in the changes in the changes in V).

For example, in the fourth row for the column labeled $\Delta^3 V$, you should enter the value of ($\Delta^2 V$ from the fourth row) – ($\Delta^2 V$ from the third row)

Math 2 **F.IF.6**

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Height of the box	Perimeter of the base of the box	ΔP	Area of the base of the box	ΔA	$\Delta^2 A$	Surface Area of the box	ΔS	$\Delta^2 S$	Volume of the box	ΔV	$\Delta^2 V$	$\Delta^3 V$
0												
1												

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7. Based on the information from your table, what would be the ***maximum value*** of each of the following:

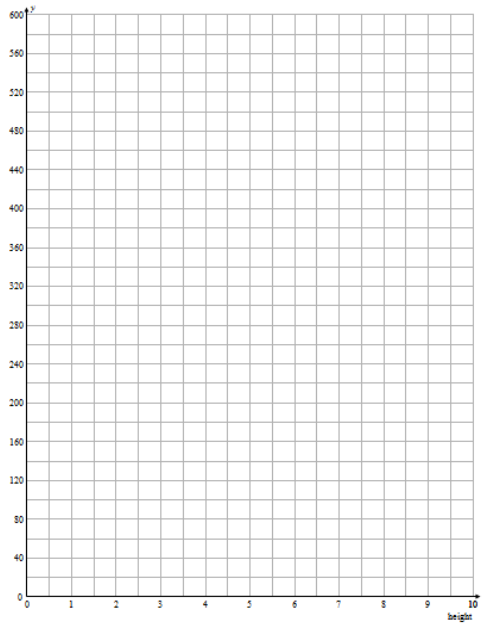
   a. the area of the base of the box: _____

   b. the surface area of the box: _____

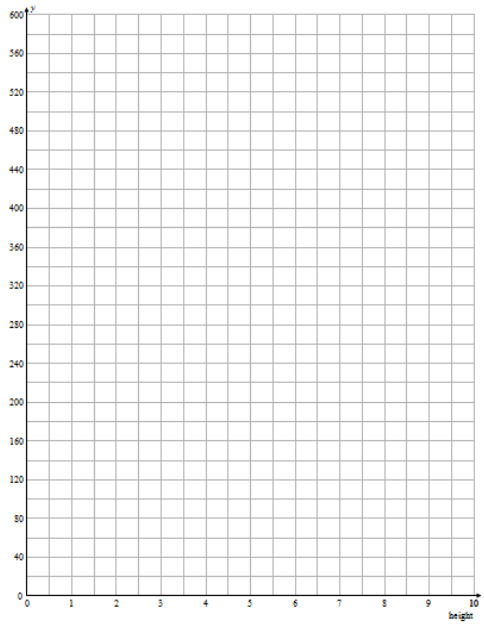
   c. the volume of the box: _____

8. Plot the following data (in four separate graphs):

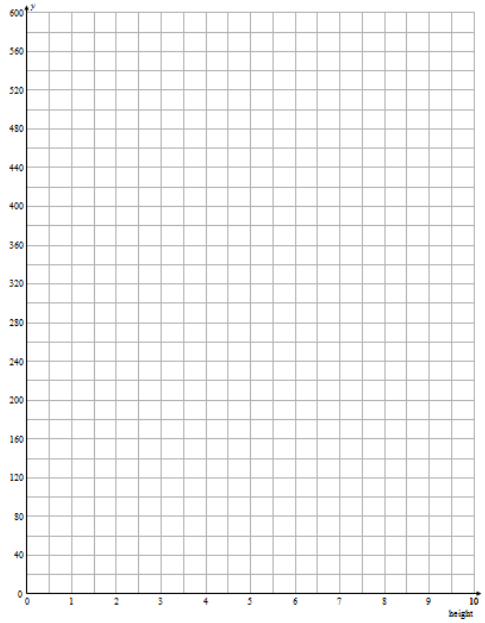
a. height of the box versus perimeter of the base of the box



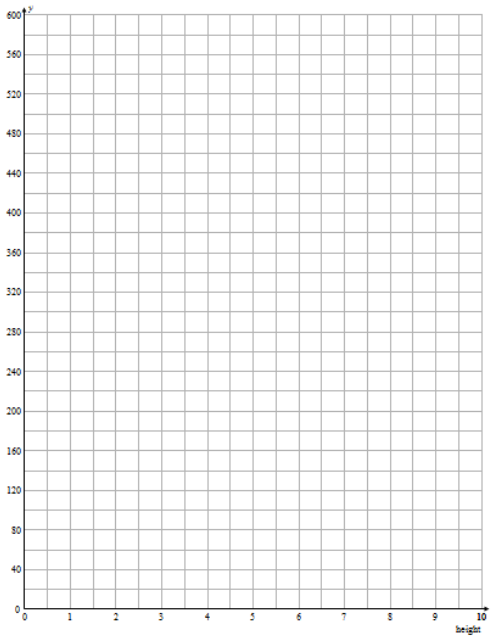
b. height of the box versus area of the base of the box



c. height of the box versus surface area of the box



d. height of the box versus volume of the box



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9. Derive a formula for each function:

   a. the perimeter, *P*, of the base of the box as a function of the height, *x*, of the box.

   $P(x) =$

   b. the area, *A*, of the base of the box as a function of the height, *x*, of the box.

   $A(x) =$

   c. the surface area, *S*, of the base of the box as a function of the height, *x*, of the box.

   $S(x) =$

   d. the volume, *V*, of the box as a function of the height, *x*, of the box.

   $V(x) =$

8. Based on the graphs you sketched above, what would be the maximum value of each of the following:

   a. the area of the base of the box: _____

   b. the surface area of the box: _____

   c. the volume of the box: _____

9. Use your calculator to find the <u>appropriate</u> regression equation for each of the following relationships. **<u>Note:</u> Since these relationships are <u>not</u> all linear, you need to use other regression equations on your calculator.**

   a. height of the box versus perimeter of the base

   b. height of the box versus area of the base

   c. height of the box versus volume of the box

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10. Explain how the regression equations you found in #9 compare to the equations you derived in #8.

11. Look at the ΔP, ΔA, $\Delta^2 A$, ΔV, $\Delta^2 V$, and $\Delta^3 V$ columns from your table. What connection, if any, can be made between these finite differences and the equations from #8 and #9?