Math 2 Unit 6 Similarity and Volume

Approximate Time Frame: 4 – 5 Weeks

Connections to Previous Learning:

In Grade 8, students were introduced to the concept of similarity through the use of physical models and dynamic geometry software. They were introduced to four types of transformations: rotations, reflections, translations, and dilations. These transformations were then used to establish whether two-dimensional figures are similar to one another. In addition, students were introduced to the Pythagorean Theorem and saw how it might be proven through informal geometric arguments. In Math 1, students developed a precise definition of congruence in terms of rigid motions, and then used this definition to determine necessary and sufficient conditions for deciding when two triangles are congruent. Here, sufficient conditions refer to triangle congruence theorems such as SSS, SAS, and ASA. Students were introduced to logical structure behind conditional statements, including the concepts of hypotheses and conclusions, and used this structure to prove a myriad of relationships between lines, angles, triangles, and parallelograms. In previous courses and in previous units of this course, students have learned and used volume formulas. In this unit, students will review and extend those applications and make connections between volume and similarity.

Focus of this Unit:

Since the concept of dilations was treated in the Grade 8 standards mostly in the context of hands-on activities, and with an emphasis on geometric intuition, one focus of this unit is to develop a more precise definition for this transformation, as was done in Math 1 for the rigid motions. The rigid motions, when combined with dilations, are referred to as similarity transformations. Specific outcomes of the unit are to develop a precise definition of similarity in terms of similarity transformations, to use this definition to develop criteria for determining when two triangles are similar, and finally, to develop triangle similarity theorems which follow from the definition of similarity in terms of similarity transformations. As in Math 1, attributes of geometric objects observed at earlier grades will now be looked at more precisely through proof; and in many instances, these proofs will make use of triangle similarity and its consequences. Key among the relationships that will be proven through the use of triangle similarity is the one found in the Pythagorean Theorem. Properties of congruent and of similar triangles will also be used to solve a variety of problems that either involve or can be modeled with triangles. Along with physical models, dynamic geometry environments will provide students with tools for investigating, experimenting with, conjecturing about, and modeling geometric phenomena. Students will experience informal arguments for determining various volume formulas and will use volume formulas to solve problems and applications. They will also learn how scaling three dimensional shapes affects their area and volume.

Connections to Subsequent Learning:

Understanding of the necessary and sufficient criteria for triangle similarity will help students not just during their investigations of the properties of lines, triangles, and quadrilaterals in this unit, but also in their future investigations of circles, in the development of right triangle trigonometry (which in turn will be used to aid in the exploration of trigonometric functions), and in many geometric modeling tasks. The definitions and properties of geometric objects considered in this unit will reappear in future units that deal with other geometric concepts, with geometry from an analytic perspective (coordinate geometry) and with modeling. Proof will continue to be a spiraled concept throughout subsequent units and courses as it will be used later on in both geometric and non-geometric settings. The problem solving techniques that will be used to solve problems from this unit will help students approach similar types of problems that they will see in many other units. In Math 3, students will be connect their understanding of volume and area to density.

Desired Outcomes

Standard(s):

Understand similarity in terms of similarity transformations.

- G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor.
- G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
- G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity.

- G.SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangles divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
- G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
  a) A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
  b) The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Explain volume formulas and use them to solve problems.

- G.GMD.1 Give an informal argument for the formula for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments. Calvalieri's principal and informal limit arguments.*
- G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Transfer:

Students will use concepts and procedures to find missing measurements in various situations using proportional relationships.

Ex. On a class trip to Ellis Island, Jose wanted to know how tall the Statue of Liberty was. He learned that at noon, the Statue of Liberty cast a 762 foot 6 inch shadow and he measured his own shadow to be 12 foot 6 inches. If Frank is 5 feet tall, how tall is the Statue of Liberty?

Ex. The photograph of the English Channel shown at the right was taken from onboard the Space Shuttle from an altitude of 203 km using a camera that has a lens with focal length of 90 mm. Calculate the shortest distance across the Channel (as marked in the photograph).

Image: NASA

WIDA Standard: (English Language Learners)

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
English language learners benefit from:

- explicit vocabulary instruction with regard to geometric properties, theorems and figures.
- tactile and virtual manipulatives for exploring the relationships (congruence and similarity) between geometric figures.

Understandings: *Students will understand that …*

- A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged, and that the dilation of a line segment is longer or shorter in the ratio given by the scale factor of the dilation.
- Two geometric figures are similar if there is a sequence of similarity transformations (dilation along with rotations, reflections, or translations) that carries one onto the other.
- Two triangles are similar if and only if corresponding pairs of angles are congruent and corresponding pairs of sides are proportional.
- It is possible to prove two triangles similar by proving that two pairs of corresponding angles of the triangles are congruent.
- Different observed relationships between geometric objects are provable using basic geometric building blocks and previously proven relationships between these building blocks and between other geometric objects.
- The geometric relationships that come from proving triangles congruent or similar may be used to prove relationships between geometric objects.
- The properties of congruent and similar triangles can be used to solve problems that either involve or can be modeled with triangles.
- Volume formulas can be used to solve applications.
- The linear ratio of similarity a/b becomes $(a/b)^2$ when comparing areas and $(a/b)^3$ when comparing volumes.

Essential Questions:

- What are the properties of dilations?
- In terms of similarity transformations, when are two geometric figures similar?
- What are the necessary conditions to know when two triangles are similar?
- What are the sufficient conditions to know that two triangles are similar?
- How can the Pythagorean Theorem be proven using the geometric relationships that come from proving triangles similar?
- How can the geometric relationships that come from proving triangles congruent or similar be applied in problems solving situations?
- Why do the formulas for circumference, area of a circle, volume of a cylinder, pyramid, and cone work?
- Can I use volume formulas to solve problems?

Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

*1. **Make sense of problems and persevere in solving them.** Students will recognize the hypothesis and conclusion in a proof statement and be able to generate the requisite proof using the given information in the proof statement, along with known facts, definitions, postulates, and theorems.

*2. **Reason abstractly and quantitatively.** Students will be able to use figures and information pertaining to a specific geometric object as an aid in reasoning about that geometric object in general.

*3. **Construct viable arguments and critique the reasoning of others.** Students will be able to create and present proofs, and be able to critique the proofs and deductive reasoning of others.

*4. **Model with mathematics.** Students will be able to solve a variety of problems that either involve or can be modeled with triangles by applying the properties of congruent and of similar triangles.

*5. **Use appropriate tools strategically.** Students will be able to use physical models, drawings, and dynamic geometry environments to form conjectures about geometric objects and to reason from information about the geometric object provided by these tools.

*6. **Attend to precision.** Students will recognize that incorrect initial attempts at definitions, conjectures, and theorems may be corrected through a process of refinement.

*7. **Look for and make use of structure.** Students will be able to use the structure of geometric objects to gain insights into, make conjectures about, and create proofs pertaining to these objects.

8. **Look for and express regularity in repeated reasoning.**

Math 2 Unit 6 Similarity and Volume

Prerequisite Skills/Concepts:	Advanced Skills/Concepts:
Students should already be able to: • Understand what constitutes a similarity transformation. • Informally understand what it means for two geometric figures to be similar. • Given an object, perform specific reflections, rotations, translations, and dilations to that object. • Identify and describe components of geometric objects such as point, line, angle, triangle, parallelogram, circle, etc. and use them to analyze geometric figures. • Recall various geometric relationships proven in previous courses. • Prove something. • Sketch triangles that model given problem situations. • Use the Pythagorean Theorem to solve problems. • Use area and volume formulas to solve problems	*Some students may be ready to:* • Recognize how counterexamples can be used to refute conjectures. • Prove further geometric theorems (relating to lines, angles, triangles, quadrilaterals, etc.). • Describe an informal proof for and use geometric formulas. • Transform graphs based on changes in equations and write equations based on a transformed parent graph.
Knowledge: *Students will know…* • The definition of various geometric objects such as angle, triangle, parallel lines, perpendicular lines, parallelogram, etc.	**Skills:** *Students will be able to …* • Recognize if one geometric object can be transformed to another through a sequence of rigid motions combined with a dilation. • Sketch a figure that represents specific given information. • Construct a conditional statement that represents a given conjecture. • Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. • Prove theorems about triangles. • Describe an informal proof for and use geometric formulas for volume.

Academic Vocabulary:

Critical Terms:	Supplemental Terms:
Similarity transformation Similar Corresponding Ratio Proportional Pythagorean Theorem Volume	Rotation Reflection Translation Dilation Conditional statement Hypothesis Conclusion Proof Necessary Conditions Sufficient Conditions Postulate Theorem

Assessment

Pre-Assessments	Formative Assessments	Summative Assessments	Self-Assessments
#1 Dilation Pre-Assessment #11 Finding volume	#2 Dilating points, lines, and planes oh my #3 Stretching for Similarity #4 Indirect Measurement #5 Traveling to Lilliput #6 Are they Similar #7 Using Similarity Criteria #8 Side Splitter theorem #9 Varignon Quadrilaterals #10 Pythagorean Theorem w/similarity #12 Calculating Volume #13 Sand Castles	#13 Sandcastles	#1 Dilation Pre-Assessment #6 Are they Similar

Sample Lesson Sequence

1. G.SRT.5 G.SRT.1,a,b, G.SRT.2- Connecting Similarity to Transformations and Congruence
    a. Review rigid motion and congruency of figures.
    b. Connect to the concept of dilation (similarity of figures).
    c. Solve problems and prove relationships between geometric figures using similarity.
2. G.SRT.3, G.SRT.4 – Using Similarity in Proofs
    a. Using similarity transformations to verify AA similarity.
    b. Using criteria for proving two triangles to be similar (AA, SAS, SSS for similarity).
    c. Use similarity to prove theorems about triangles including:
        i. Side-Splitter Theorem
        ii. Pythagorean theorem with triangle similarity
3. G.GMD1, G.GMD.3 – Using Volume
    a. Provide informal arguments as to why select geometric formulas make sense.
    b. Use volume formulas to solve problems.
    c. Connecting scaling volume to similarity.