Math 2 Unit 4 Quadratic Functions: Modeling

Approximate Time Frame: 2 – 3 Weeks

Connections to Previous Learning:

In Math 1, students learn how to model functions and how to interpret models of functions. Students looked at multiple representations of linear and exponential functions. In previous units, students learned the structure of a quadratic expression, equation, and function, different techniques for solving quadratic equations, and the relationship between the solution of a quadratic equation and the zeros of a quadratic function.

Focus of this Unit:

This unit focuses on graphing and modeling quadratic functions. Students can use different techniques for solving quadratic equations, including factoring and completing the square, to graph the corresponding quadratic function and to show key features such as zeros, extreme values, and symmetry. Students can rewrite quadratic equations in equivalent forms to reveal new information about the function and its graph to model real-life situations.

Connections to Subsequent Learning:

Students will take their new knowledge of graphing and modeling quadratic functions and apply it to creating quadratic functions that model physical phenomena. Students will extend their modeling skills to Polynomial, Rational, Logarithmic, and Trigonometric Functions. Students will continue to study the general shape of functions, the parent function, how the function can be transformed and how equivalent equations can reveal new information about the function.

From the Grade 8, High School, Functions Progression Document pp. 8-9:

Interpret functions that arise in applications in terms of the context.

Functions are often described and understood in terms of their *behavior*: Over what input values is it increasing, decreasing, or constant? For what input values is the output value positive, negative, or 0? What happens to the output when the input value gets very large positively or negatively? Graphs become very useful representations for understanding and comparing functions because these “behaviors” are often easy to see in the graphs of functions (see illustration). Graphs and contexts are opportunities to talk about domain (for an illustration, go to illustrativemathematics.org/illustrations/631).

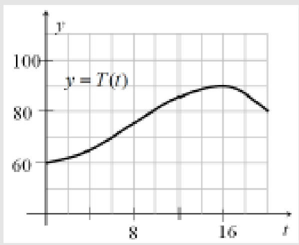
Graphs help us reason about rates of change of function. Students learned in Grade 8 that the rate of change of a linear function is equal to the slope of its graph. And because the slope of a line is constant, the phrase “rate of change” is clear for linear functions. For nonlinear functions, however, rates of change are not constant, and so we talk about average rates of change over an interval. For example, for the function $g(x) – x^2$, the average rate of change from x = 2 to x = 5 is

$$\frac{g(5)-g(2)}{5-2} = \frac{25-4}{5-2} = \frac{21}{3} = 7$$

This is the slope of the line from (2, 4) to (5, 25) on the graph of *g*. And if *g* is interpreted as returning the area of a square of side x, then this calculation means that over this interval the area changes, on average, 7 square units for each unit increase in the side length of the square.

Warming and Cooling

The figure shows the graph of T, the temperature (in degrees Fahrenheit) over one particular 20-hour period in Santa Elena as a function of time t.



(a) Estimate $T(14)$.

(b) If $t = 0$ corresponds to midnight, interpret what we mean by $T(14)$ in words.

(c) Estimate the highest temperature during this period from the graph.

(d) When was the temperature decreasing?

(e) If Anya wants to go for a two-hour hike and return before the temperature gets over 80 degrees, when should she leave?

For solutions and more discussion of this task, go to Illustrative Mathematics at illustrativemathematics.org/illustrations/639.

Major Standards Supporting Standards Additional Standards

From the Grade 8, High School, Functions Progression Document p. 11:

Build a function that models a relationship between two quantities.

This cluster of standards is very closely related to the algebra standard on writing equations in two variables. Indeed, that algebra standard might well be met by a curriculum in the same unit as this cluster. Although students will eventually study various families of functions, it is useful for them to have experiences of building functions from scratch, without the aid of a host of special recipes, by grappling with a concrete context for clues. For example, in the Lake Algae task, parts (a)-(c) lead students through reasoning that allows them to construct the function in part (d) directly. Students who try a more conventional approach in part (d) of fitting the general function $f(t) = ab^t$ to the situation might get confused or replicate work already done.

The Algebra Progression discusses the difference between a function and an expression. Not all functions are given by expressions, and in many situations it is natural to use a function defined recursively. Calculating mortgage payment and drug dosages are typical cases where recursively defined functions are useful (see example).

Lake Algae

On June 1, a fast growing species of algae is accidentally introduced into a lake in a city park. It starts to grow and cover the surface of the lake in such a way that the area covered by the algae doubles every day. If it continues to grow unabated, the lake will be totally covered and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30.

(a) When will the lake be covered half-way?

(b) On June 26, a pedestrian who walks by the lake every day warns that the lake will be completely covered soon. Her friend just laughs. Why might her friend be skeptical of the warning?

(c) On June 29, a clean-up crew arrives at the lake and removes almost all of the algae. When they are done, only 1% of the surface is covered with algae. How well does this solve the problem of the algae in the lake?

(d) Write an equation that represents the percentage of the surface area of the lake that is covered in algae as a function of time (in days) that passes since the algae was introduced into the lake if the cleanup crew does not come on June 29.

For solutions and more discussion of this task, go to Illustrative Mathematics at illustrativemathematics.org/illustrations/533.

Drug Dosage

A student strained her knee in an intramural volleyball game, and her doctor has prescribed an anti-inflammatory drug to reduce the swelling. She is to take two 220-milligram tablets every 8 hours for 10 days. Her kidneys filter 60% of this drug from her body every 8 hours. How much of the drug is in her system after 24 hours?

Task from High School Mathematics at Work: Essays and Examples for the Education of All Students (1998), National Academies Press. See http://www.nap.edu/openbook/0309063531/html/80.html for a discussion of the task.

Modeling contexts also provide a natural place for students to start building functions with simpler functions as components. Situations of cooling or heating involve functions which approach a limiting value according to a decaying exponential function. Thus, if the ambient room temperature is 70° and a cup of tea made with boiling water at a temperature of 212°, a student can express the function describing the temperature as a function of time using the constant function *f(t)*=70 to represent the ambient room temperature and the exponentially decaying function $g(t) = 142e^{-kt}$ to represent the decaying difference between the temperature of the tea and the temperature of the room, leading to a function of the form

$$T(t) = 70 + 142e^{-kt}.$$

Students might determine the constant *k* experimentally. In contexts where change occurs at discrete intervals (such as payments of interest on a bank balance) or where the input variable is a whole number (for example the number of a pattern in a sequence of patterns), the functions chosen will be sequences.

Desired Outcomes

Standard(s):

Create equations that describe numbers or relationships.

- A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Interpret functions that arise in applications in terms of the context.

- F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory then the positive integers would be an appropriate domain for the function.
- F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Build a function that models a relationship between two quantities.

- F.BF.1 Write a function that describes a relationship between two quantities.
  a) Determine an explicit expression, a recursive process, or steps for calculation from a context.
  b) Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Transfer:

Students will be given a real-world situation and write a quadratic function to model that situation, manipulate the function to find new information about the situation, and use multiple representations of the function to answer questions.

Ex. A video store rents 1750 videos per week at $3.25 per video. The owner estimates that they will rent 150 fewer videos for each $0.25 increase in price. What price will maximize the income of the video store?

Ex. A student drops a ball from the top of a building and records the height of the ball at different times, as shown in the table to the right.

Time (s)	Height (ft)
0	46
1	63
2	48
3	1

- Find a quadratic model for the data.
- Use the model to estimate the height of the ball at 1.5 seconds.
- What is the ball's maximum height?

Students will apply concepts and procedures regarding solving quadratic equations and their multiple representations in order to interpret key features of the graph of the corresponding quadratic function.

Example: Given a quadratic equation, students will solve the equation and use this information to identify the intercepts and vertex of the graph of the corresponding quadratic function.

Understandings: *Students will understand that…*

- Quadratic expressions have equivalent forms that can reveal new information to aid in graphing quadratic functions and solving problems.
- Quadratic functions have key features that can be represented on a graph and can be interpreted to provide information to describe relationships of two quantities.
- A quadratic function has a domain that provides information to the function, graph, and situation that it describes.
- The average rate of change can be estimated, calculated, or analyzed from a quadratic function or a graph.
- Quadratic functions, like linear and exponential, can be used to model real-life situations. These equations can be represented in multiple ways to reveal new information.

Essential Questions:

- How can key features of a quadratic expression be used to generate the graph of the quadratic function corresponding to that expression?
- What new information about the graph of a quadratic function will be revealed if the quadratic function is written in a different but equivalent form?
- What do the key features of a quadratic graph represent in a modeling situation?
- How do you create an appropriate function to model data or situations given within context?

Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

1. **Make sense of problems and persevere in solving them.**

*2. **Reason abstractly and quantitatively**. Students will use given information to determine what form of a quadratic to use for modeling. For example, a set of data that appears to have a minimum value might lead a student to choose vertex form over standard form of a quadratic. Students will solve problems and interpret solutions considering scale, units, data displays, and levels of accuracy.

3. **Construct viable arguments and critique the reasoning of others.**

*4. **Model with mathematics.** Students will use graphs, tables, and equations to model quadratic equations.

*5. **Use appropriate tools strategically.**

*6. **Attend to precision**. Students will use appropriate scales and levels of precision in their models and predictions, as determined by the precision in the data.

7. **Look for and make use of structure.**

8. **Look for and express regularity in repeated reasoning.**

Prerequisite Skills/Concepts: *Students should already be able to:*	**Advanced Skills/Concepts:** *Some students may be ready to:*
• Solve one-variable equations and recognize equivalent forms. • Model real-world one-variable equations and two-variable equations limited to linear and exponential. • Define a function and distinguish between coefficients, factors, and terms. • Recognize how functions can be represented on a graph and in a table and how scale and labels can modify the appearance of the representation. • Represent real-world situations with a linear or exponential model and make decisions about the appropriateness of each. • Solve a variety of quadratic equations using different methods including taking square roots, factoring, completing the square, and using the quadratic formula.	• Compose and decompose functions to create and interpret models. For example, interpret the meaning of (x+2) in $y = (x+2)^2 – 3(x+2) -5$. • Analyze the appropriateness of a quadratic model. • Compare quadratic and catenary models for given physical phenomena. • Use finite differences to determine the appropriateness of a quadratic model.
Knowledge: *Students will know…* • The general shape of the graph of a quadratic function. • The sign of the leading coefficient determines the direction that the parabola opens. • The different information about the graph of a quadratic function that can be found in different but equivalent forms of the function	**Skills:** *Students will be able to …* • Write a quadratic equation and/or function to model a real-life situation. • Use a quadratic model to interpret information about physical phenomena. • Translate among representations of quadratic functions including tables, graphs, equations, and real-life situations. • Rewrite quadratic functions to reveal new information. • Sketch the graph of a quadratic function based upon its symbolic form. • Estimate, calculate, and interpret the average rate of change over a specified interval of a quadratic. • Determine an explicit expression, a recursive process, or steps for calculation from a context. • Combine standard function types using arithmetic operations.

WIDA Standard: (English Language Learners)

English language learners communicate information, ideas, and concepts necessary for academic success in the content area of Mathematics.
English language learners benefit from:

- Explicit vocabulary instruction with regard to components of equations and features of graphs.
- Explicit instruction regarding the connections between mathematical representations and real-world contexts.

Academic Vocabulary:

Critical Terms:	Supplemental Terms:
	Quadratic Parabola Completing the Square Quadratic Formula Standard form Vertex form Intercepts Intervals relative maximums relative minimums symmetries end behavior periodicity

Assessments

Pre-Assessments	Formative Assessments	Summative Assessments	Self-Assessments
2 Pre-Assessment for Sample LP	*1 Maximizing Area* *4 Quadratic Gallery Walk* *3 Box Problem*		*2 Pre Assessment for Sample LP*

Sample Lesson Sequence:

1. A.CED.2, F.BF.1 Using quadratic equations to model situations
    a. Finding solutions of quadratic equations by inspecting a graph or table in the context of modeling
    b. Writing a quadratic function using information from a graph, table, or story problem
    c. Graphing quadratic functions from a context or situation

2. **F.IF.4, F.IF.5, 6 Looking at the graphs of quadratic functions that model real life situations – Sample Lesson Plan**
    a. **Obtaining information from graphs**
        i. **Calculating rate of change of a quadratic and relating it to a given situation**
        ii. **Interpreting maximum and minimum in context**
    b. **Building connections between the graph of a quadratic function and the situation it is modeling (i.e. what does the x-intercept represent in the context of the problem?).**