Time Frame: Approximately 3-5 weeks

Connections to Previous Learning:

Students build upon previous understandings of linear equations and functions and apply them to various representations of linear relationships, including tables, graphs, and equations.

From the Grade 8, High School, Functions Progression Document, pp. 7-9:

Interpreting Functions

Understand the concept of a function and use function notation Building on semi-formal notions of functions from Grade 8, students in high school begin to use formal notation and language for functions. Now the input/output relationship is a correspondence between two sets: the domain and the range. The domain is the set of input values, and the range is the set of output values. A key advantage of function notation is that the correspondence is built into the notation. For example, *f* (5) is shorthand for "the output value of *f* when the input value is 5."

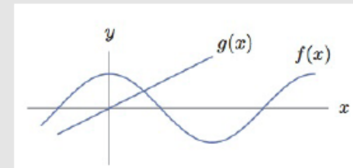
Students sometimes interpret the parentheses in function notation as indicating multiplication. Because they might have seen numerical expressions like 3(4), meaning 3 times 4, students can interpret *f(x)* as *f* times *x*. This can lead to false generalizations of the distributive property, such replacing *f(x+* 3) with *f(x)* +*f*(3). Work with interpreting function notation in terms of the graph of *f* can help students avoid this confusion with the symbols (see example to right).

Although it is common to say "the function *f(x)*," the notation *f(x)* refers to a single output value when the input value is *x*. To talk about the function as a whole, write *f,* or perhaps "the function f, where *f(x)* =3*x*+ 4." The *x* is merely a placeholder, so *f (t)* 3*t*+ 4 describes exactly the same function. Later, students can make interpretations like those in the following table:

Expression	Interpretation
$f(a+2)$	The output when the input is 2 greater than a
$f(a)+3$	3 more than the output when the input is a
$2f(x)+5$	5 more than twice the output of f when the input is x
$f(b)-f(a)$	The change in output when the input changes from a to b

Interpreting the graph

Use the graph (for example, by marking specific points) to illustrate the statements in (a)–(d). If possible, label the coordinates of any points you draw.



(a) $f(0) = 2$

(b) $f(3) = f(3) = f(9) = 0$

(c) $f(2) = g(2)$

(d) $g(x) > f(x)$ for $x > 2$

For solutions and more discussion of this task, go to Illustrative Mathematics at illustrativemathematics.org/illustrations/636.

The square root function

Since the equation $x^2 = 9$ has two solutions, $x = \pm 3$, students might think incorrectly that $\sqrt{9} = \pm 3$. However, if we want $\sqrt{x}$ to be a function of x, we need to choose one of these square roots. The square root function, $g(x) = \sqrt{x}$, is defined to be the positive square root of x for any positive x.

Notice that a common preoccupation of high school mathematics distinguishing function from relations is not in the Standards. Time normally spent on exercises involving the vertical line test, or searching lists of ordered pairs to find two with the same *x*-coordinate and different *y*-coordinate, can be reallocated elsewhere. Indeed, the vertical line test is problematical, since it makes it difficult to discuss questions such as "is *x* a function of *y*" when presented with a graph of *y* against *x* (an important question for students thinking about inverse functions). The core question when investigating functions is: "Does each element of the domain correspond to exactly one element in the range?" The graphic on the next page shows a discussion of the square root function oriented around this question.

Cell Phones

Let $f(t)$ be the number of people, in millions, who own cell phones t years after 1990. Explain the meaning of the following statements.

(a) $f(10) = 100.3$

(b) $f(a) = 20$

(c) $f(20) = b$

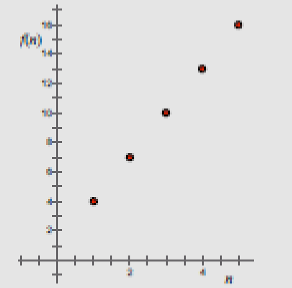
(d) $n = f(t)$

For solutions and more discussion of this task, go to illustrativemathematics.org/illustrations/634.

To promote fluency with function notation, students interpret function notation in contexts. For example, if *h* is a function that relates Kristin's height in inches to her age in years, then the statement *h*(7) = 49 means, "When Kristin was 7 years old, she was 49 inches tall." The value of *h*(12) is the answer to "How tall was Kristin when she was 12 years old." And the solution of *h(x)*= 60 is the answer to "How old was Kristen when she was 60 inches tall?"

Sequences as functions

n	$f(n)$
1	4
2	7
3	10
4	13
5	16



Sometimes, especially in real-world contexts, there is no expression (or closed formula) for a function. In those cases, it is common to use a graph or a table of values to (partially) represent the function.

A *sequence* is a function whose domain is a subset of the integers. In fact; many patterns explored in grades K-8 can be considered sequences. For example, the sequence 4, 7, 10, 13, 16 . . . might be described as a "plus 3 pattern" because terms are computed by adding 3 to the previous term. To show how the sequence can be considered a function, we need an *index* that indicates which term of the sequence we are talking about, and which serves as an input value to the function. Deciding that the 4 corresponds to an index value of 1, we make a table showing the correspondence, as in the margin. The sequence can be describe recursively by the rule $f(1) = 4$, $f(n + 1) = f(n) + 3$ for ≥ 2. Notice that the recursive definition requires both a starting value and a rule for computing subsequent terms. The sequence can also be described with the closed formula $f(n) = 3n + 1$, for integers $n \geq 1$. Notice that the domain is included as part of the description. A graph of the sequence consists of discrete dots, because the specification does not indicate what happens "between the dots."

+ In advanced courses, students may use subscript notation for sequences.

Analyze functions using different representations. Functions are often studied and understood as families, and students should spend time studying functions within a family, varying parameters to develop an understanding of how the parameters affect the graph of function and its key features.

Within a family, the functions often have commonalities in the qualitative shapes of their graphs and in the kinds of features that are important for identifying functions more precisely within a family. This standard indicates which function families should be in students' repertoires, detailing which features are required for several key families. It is an overarching standard that covers the entire range of a student's high school experience; in this part of the progression we merely indicate some guidelines for how it should be treated.

First, linear and exponential functions (and to a lesser extent quadratic functions) receive extensive treatment and comparison in a dedicated group of standards, **Linear and Exponential Models**. Thus, those function families should receive the bulk of the attention related to this standard. Second, all students are expected to develop fluency with linear, quadratic, and exponential functions, including the ability to graph them by hand. Finally, in most of the other function families, students are expected to graph simple cases without technology, and more complex ones with technology.

Desired Outcomes

Standard(s):

Understand the concept of a function and use function notation.

- F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1)$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*

Build a function that models a relationship between two quantities.

- F.BF.2 Write arithmetic ~~and geometric~~ sequences both recursively and with an explicit formula, use them to model situations and translate between two forms.

Represent and solve equations and inequalities graphically.

- A.REI.10 Understand that the graph of an equation in two variable is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Create equations that describe numbers or relationship.

- A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear *~~and quadratic functions, and simple rational and exponential~~ functions.**
- A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equation on coordinate axes with labels and scales.
- A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reason as in solving equations. For example, *rearrange Ohm's law $v = IR$ to highlight resistance R.*

Interpret functions that arise in applications in terms of the context.

- F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
- F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

- F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases using technology for more complicated cases.*
  a) Graph linear ~~and quadratic functions~~ and show intercepts, ~~maxima, and minima~~.
  b) Graph ~~square root, cube root and~~ piecewise-defined functions, including step functions and absolute value functions.

*Elements of this standard that are crossed out are not addressed in this unit and will be addressed at a different time.

Transfer: Students will be able to develop concepts and procedures of representing linear relations to analyze functions.

- Given the equation y = ¼x + 12 a student can represent it as a graph, situation, or table.

WIDA Standard: (English Language Learners)

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
English language learners benefit from:

- explicit vocabulary instruction with regard to key features of graphs and tables.
- explicit instruction with regard to the relationship between graphs or tables and the equations and contexts they represent.
- explicit instruction with regard to connecting mathematical representations to the context of situations.

Understandings: *Students will understand that …*

- Linear relationships have a constant rate of change.
- The graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which are points that either lie along a line (discrete) or form a line (continuous).
- Arithmetic sequences are functions with a domain that is a subset of the integers and can be identified by the constant difference between consecutive terms.
- Arithmetic sequences follow a discrete linear pattern, and the common difference is the slope of the line.
- Linear functions can be represented by a table, graph, verbal description or equation and that each representation can be transferred to another representation.

Essential Questions:

- What are the characteristics of a linear function?
- What is an arithmetic sequence and how does it relate to linear functions?
- What is the relationship between recursive and explicit equations and how are they represented symbolically?
- How can we represent a linear function?

Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**

*3. **Construct viable arguments and critique the reasoning of others.** Students explain why representations are linear or non-linear relationships, focusing on constant rate of change.

*4. **Model with mathematics.** Students will model the relationship between two quantities by building a function that represents an arithmetic sequence.

5. **Use appropriate tools strategically.** Students use technology tools to graph functions and analyze relationships.

6. **Attend to precision.**

*7. **Look for and make use of structure.** Students use the structure of linear relationships to recognize and build arithmetic sequences and determine rates of change from various representations of linear functions.

*8. **Look for and express regularity in repeated reasoning.** Students notice that the common difference in arithmetic sequences is the slope of the graph of the linear equation and serves as its explicit definition.

Prerequisite Skills/Concepts:

Student should already be able to:

- Define a function as a rule that assigns to each input exactly one output, and that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line.
- Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
- Understand the definitions of function, domain, and range, and function notation $y=f(x)$.
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Advanced Skills/Concepts:

Some students may be ready to:

- Recognize and create situations that would be modeled by piecewise linear or absolute value functions.

Knowledge: *Students will know…*

All standards in this unit go beyond the knowledge level.

Skills: *Students will be able to …*

- Determine the slope of a linear relationship using two points (table or graph).
- Determine the slope of a linear relationship using its equation.
- Complete a table of linear data.
- Create linear equations and inequalities in one variable and use them to solve problems.
- Create equations in two or more variables to represent relationships between quantities.
- Graph an equation on coordinate axes with labels and scales.
- Rearrange formulas to highlight a quantity of interest, using the same reason as in solving equations.
- Determine the common difference in an arithmetic sequence.
- Write both recursive and explicit equations of arithmetic sequences.
- Translate among representations of linear functions including tables, graphs, equations and real-life situations.
- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities.

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Sketch graphs showing key features given a verbal description of the relationship.
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.
Estimate the rate of change from a graph.

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<tr><th colspan="4">Academic Vocabulary:</th></tr>
<tr><td colspan="2">Critical Terms:
Domain
Term
Arithmetic sequence
Recursive formula
Explicit formula
Common difference
Linear relation/function
Domain
Continuous
Discrete</td><td colspan="2">Supplemental Terms:
Integers
Function
Rate of change
Slope
y-intercept</td></tr>
<tr><th colspan="4">Assessment</th></tr>
<tr><th>Pre-Assessments</th><th>Formative Assessments</th><th>Summative Assessments</th><th>Self-Assessments</th></tr>
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<tr><th colspan="4">Sample Lesson Sequence</th></tr>
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