Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

Targeted Content Standard(s):

7. NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a) Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers, interpret products of rational numbers by describing real-world contexts.

b) Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then *–(p/1) = (-p)/q = p/(-q).* Interpret quotients of rational numbers by describing real-world contexts.

Student Friendly Learning Targets

I can…

- Multiply and divide rational numbers
- Identify and apply properties when multiplying and dividing rational numbers

Targeted Mathematical Practice(s):

- ☒ 1 Make sense of problems and persevere in solving them
- ☒ 2 Reason abstractly and quantitatively
- ☒ 3 Construct viable arguments and critique the reasoning of others
- ☐ 4 Model with mathematics
- ☐ 5 Use appropriate tools strategically
- ☒ 6 Attend to precision
- ☐ 7 Look for and make use of structure
- ☐ 8 Look for an express regularity in repeated reasoning

Supporting Content Standard(s): *(optional)*

7.EE 2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Purpose of the Lesson:

Students will be able to multiply and divide integers fluently as well as solve real-world and mathematical problems. Students will represent their calculations using number line models, equations, expressions, and real-world applications.

Explanation of Rigor: *(Fill in those that are appropriate.)*

Conceptual:	**Procedural:**	**Application:**
Students extend previous conceptual understandings of operations to signed numbers. Students represent multiplication and division of integers with manipulatives, number lines, and other visual models. Students understand that integers cannot be divided by zero. (7.NS.2ab)	Students multiply integers fluently after interpreting multiplication patterns observed when using manipulatives. (7.NS.2ab)	Students interpret products and quotients of integers by describing real-world contexts. Students also interpret real world contexts involving positive and negative values by representing the situations with integer expressions and equations. (7.NS.2ab)

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

Vocabulary:

commutative property, distributive property, integers, negative numbers, opposites, positive numbers, income/profit, increase, decrease, number line diagram

Evidence of Learning (Assessment):

Pre-Assessment: Interpreting Absolute Value (From previous lesson)

Formative Assessment(s): Integer Multiplication Practice, Integer Division Practice, Salute

Summative Assessment: Problem Solving

Self-Assessment: Problem Solving Self-Assessment Skeleton

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

Lesson Procedures:

Segment 1

Approximate Time Frame: 30-40 minutes	**Lesson Format:** ☒ Whole Group ☐ Small Group ☒ Independent ☒ Modeled ☒ Guided ☐ Collaborative ☐ Assessment	**Resources:** 4 Operations
Focus: Students understand how the 4 operations are interrelated.		**Modalities Represented:** ☒ Concrete/Manipulative ☒ Picture/Graph ☐ Table/Chart ☐ Symbolic ☒ Oral/Written Language ☐ Real-Life Situation

Math Practice Look For(s): **MP1**: Students explain and demonstrate understanding of multiplying signed integers. **MP6:** Students demonstrate precision by using correct terminology and checking for reasonableness.	**Differentiation for Remediation:** **Differentiation for English Language Learners:** **Differentiation for Enrichment:**
Potential Pitfall(s):	**Independent Practice (Homework):**

Steps: 1. Have students break into groups of 2-3 and give them the 4 Operations graphic. Students should discuss the relationships between the operations while one student records their thoughts bout the graphic. As students discuss and explain the reasoning for the connections they make, they may use manipulatives and/or visuals to support their reasoning.	**Teacher Notes/Reflections:**
2. Have students speak out regarding the conversations they had with their groups.	**Teacher Notes/Reflections:**
Consider the following questions to facilitate discussion around these relationships: • What are the relationships between these operations? • What are some situations where you: • Add? • Subtract? • Multiply? • Divide? • Use two operations?	**Teacher Notes/Reflections:**

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

• Use negative numbers? • Post graphic of number line with repeated addition of '4 + 4 + 4'. Ask, "What operations do you see?" • Post '-4 + -4 + -4'. Ask, "What operations do you see?" • Post a table. What operations are evident?	

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

Segment 2

Approximate Time Frame: 60-80 minutes	**Lesson Format:** ☒ Whole Group ☒ Small Group ☐ Independent ☐ Modeled ☒ Guided ☒ Collaborative ☐ Assessment	**Resources:** Integer Multiplication Practice
Focus: Representing multiplication of integers with manipulatives, visuals, language, and equations.		**Modalities Represented:** ☒ Concrete/Manipulative ☒ Picture/Graph ☐ Table/Chart ☒ Symbolic ☒ Oral/Written Language ☒ Real-Life Situation

Math Practice Look For(s):

MP1: Students explain and demonstrate rational number operations by using symbols, visuals, words, and real-life contexts. Students demonstrate perseverance while using a variety of strategies (number lines, manipulatives, drawings, etc.).

MP2: Students demonstrate quantitative reasoning by representing and solving real world situations using visuals, numbers, and symbols. Students demonstrate abstract reasoning by translating numerical sentences into real -world situations.

Differentiation for Remediation:

Differentiation for English Language Learners:

Differentiation for Enrichment:

Potential Pitfall(s):

Independent Practice (Homework):

Build It, Draw It, Solve It, Explain It - Multiplication

Steps:

Portions of this segment are adapted from: http://www.erusd.k12.ca.us/ProjectAlphaWeb/index_files/NS/Using%20Integer%20Tiles%20-%20Multiply%20&%20Divide.pdf

Students should already know how to represent integers using colored chips based on their experience with adding and subtracting integers.

1. Students will represent multiplication of integers as repeated addition. Begin with repeated addition of positive integers:

When multiplying integers:

- The sign of the first value tells us if we're adding or removing groups.
- The sign of the second value tells us which chips are in the groups.

Teacher Notes/Reflections:

Example: (+3) • (+4) reads, "Add 3 groups of 4 positive chips". (+ + + +) (+ + + +) (+ + + +) Therefore, (+3) • (+4) = +12	
2. If you have to "remove" groups of chips you don't have, you could add as many zero-pairs as needed until you have enough tiles. Example: (-3) • (+4) reads, "Remove 3 groups of 4 positive chips". Add at least 12 zero-pairs: + + + + + + + + + + + + (- - - - - - - - - - - -) Remove 3 groups of 4 positive chips. (+ + + +) (+ + + +) (+ + + +) Therefore, (-3) • (+4) = -12 Once students remove the groups of positive chips, they should see that 12 negative chips remain.	**Teacher Notes/Reflections:**
3. When mixed operations are shown, the language applied to the situation is essential to understanding the actions. Example: (+3) • (-4) reads, "Add 3 groups of 4 negative chips". Students add the 3 groups of negative chips and can use repeated addition to comprehend that 12 negative chips are shown. (- - - -) (- - - -) (- - - -) Therefore, (+3) • (-4) = -12	**Teacher Notes/Reflections:**
4. (-3) • (-4) reads, "Take away 3 groups of 4 negative tiles". Add at least 12 zero-pairs: + + + + + + + + + + + + (- - - - - - - - - - - -) Remove the 3 groups of 4 negative chips. (- - - -) (- - - -) (- - - -) Therefore, (-3) • (-4) = +12 Once students remove the groups of negative chips, they should see that 12 positive chips remain.	**Teacher Notes/Reflections:**
5. Students continue to make sense of the connection between operations on integers and language, representing their quantities with the 2-color chips. Give students the Integer Multiplication Practice page. Students write the language used to describe the equations, build the models with 2-color chips, record the models with + and – signs, and find the products.	**Teacher Notes/Reflections:**
6. Provide students further practices multiplying integers using the Build It, Draw It, Solve It, Explain It page. Students choose 2 cards from a deck of cards. (Pull out the Jacks, Queens and Kings.) Reds are negative numbers and blacks are positive numbers. Students	**Teacher Notes/Reflections:**

model the expressions visually (chips, diagrams, etc.) and record the results of the calculations.	
7. Students play the game Salute in groups of 3 to practice their integer multiplication concepts. 2 students are playing against each other to find their missing factor first. The 3rd player is the moderator. The moderator tells the 2 players to "salute" by holding a card up to their forehead. They show each other the cards but cannot see the card held to their head. The moderator mentally computes the product of the 2 cards and states the product aloud. Each player tries to figure out their card first. Whoever states their card correctly first, captures both cards for their win pile. If there is a tie, the moderator holds the cards until the next round is won. That winner also captures the tie pile. *(From Box Cars and One-Eyed Jacks)*	

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

Segment 3

Approximate Time Frame:	**Lesson Format:**	**Resources:**
40-60 minutes	☒ Whole Group ☐ Small Group ☒ Independent	
Focus: Analyze the concept of opposites using manipulatives, number line models, real-life situations, and language.	☒ Modeled ☒ Guided ☒ Collaborative ☐ Assessment	**Modalities Represented:** ☒ Concrete/Manipulative ☒ Picture/Graph ☒ Table/Chart ☒ Symbolic ☒ Oral/Written Language ☒ Real-Life Situation

Math Practice Look For(s):

MP1: Students explain and demonstrate the concept of opposites by using symbols, visuals, words, and real-life contexts. Students demonstrate perseverance while using a variety of strategies (number lines, manipulatives, drawings, etc.).

MP2: Students demonstrate quantitative reasoning by representing and solving real-world situations using visuals, numbers, and symbols. They demonstrate abstract reasoning by translating numerical sentences into real - world situations.

Differentiation for Remediation:

Differentiation for English Language Learners:

Differentiation for Enrichment:

Potential Pitfall(s):

Independent Practice (Homework):

Steps:

1. Pose the following question to students:

   "How have you been modeling the term "opposite" in your work with multiplying integers?"

   Give students 2-3 minutes to write about this question as a journal entry. Then, have them share their thoughts with a partner or small group before asking students to share their thoughts with the whole group.

   Hopefully, some students have come to the conclusion that positive and negative numbers show "opposite" as well as the terms "add" and "remove." If not, the teacher may need to ask more supporting questions to aid students in reaching these conclusions. ("What actions did you do with the chips?", " How are those actions different?", etc.)

Teacher Notes/Reflections:

Essential Question:

- What is the relationship between properties of operations and types of numbers?

2. Ask students how they might model multiplication of integers on a number line diagram. Have them work independently for 2-3 minutes before sharing in their small group. Debrief with the whole group by asking students how their representations were similar and how they were different. Students may share their work on the board.	**Teacher Notes/Reflections:**
3. Have student create three different diagrams that model integer multiplication. Then, use precise language to describe real-world situations represented by the diagrams.	**Teacher Notes/Reflections:**

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

Segment 4

Approximate Time Frame: 60-80 minutes	**Lesson Format:** ☒ Whole Group ☒ Small Group ☒ Independent	**Resources:** Integer Division Practice
Focus: Representing division of integers with manipulatives, visuals, language, and equations.	☒ Modeled ☒ Guided ☒ Collaborative ☐ Assessment	**Modalities Represented:** ☒ Concrete/Manipulative ☒ Picture/Graph ☒ Table/Chart ☒ Symbolic ☒ Oral/Written Language ☒ Real-Life Situation

Math Practice Look For(s):

MP1: Students explain and demonstrate rational number operations by using symbols, visuals, words, and real-life contexts. Students demonstrate perseverance while using a variety of strategies (number lines, manipulatives, drawings, etc.).

MP2: Students demonstrate quantitative reasoning by representing and solving real world situations using visuals, numbers, and symbols. They demonstrate abstract reasoning by translating numerical sentences into real - world situations.

Differentiation for Remediation:

Differentiation for English Language Learners:

Differentiation for Enrichment:

Potential Pitfall(s):

Independent Practice (Homework):

Build It, Draw It, Solve It, Explain It - Division

Steps:

Portions of this segment are adapted from:
http://www.erusd.k12.ca.us/ProjectAlphaWeb/index_files/NS/Using%20Integer%20Tiles%20-%20Multiply%20&%20Divide.pdf

Dividing Integers:

After exploring multiplication using integer tiles, dividing integers could be explained as the inverse of multiplying integers.

1. Students will represent division of integers as repeated subtraction. For example: If 3 • 5 = 15 then the inverse would be 15 ÷ 5 = 3 and vice-versa.
Therefore, any division problem could be solved if it's represented as the inverse of amultiplication problem.
Examples:
(+6) ÷ (+3) = (?) rewrites as: (?) • (+3) = (+6)

Teacher Notes/Reflections:

2. How many groups of 3 positive tiles would you have to add/take away to get 6 positive tiles? (+ + +) (+ + +) You would have to add 2 groups of 3 positive tiles. ? = +2 Therefore, (+6) ÷ (+3) = (+2) (+6) ÷ (-3) = (?) rewrite as: (?) • (-3) = (+6)	
3. How many groups of 3 negative tiles would you have to add/take away to get 6 positive tiles? (+ + +) (+ + +) You would have to take away 2 groups of 3 negative tiles. ? = -2 Therefore, (+6) ÷ (-3) = (-2) (-6) ÷ (+3) = (?) rewrite as: (?) • (+3) = (-6)	**Teacher Notes/Reflections:**
4. How many groups of 3 positive tiles would you have to add/take away to get 6 negative tiles? (+ + +) (+ + +) You would have to take away 2 groups of 3 positive tiles. ? = -2 Therefore, (-6) ÷ (+3) = (-2) (-6) ÷ (-3) = (?) rewrite as: (?) • (-3) = (-6)	**Teacher Notes/Reflections:**
5. How many groups of 3 negative tiles would you have to add/take away to get 6 negative tiles? (- - -) (- - -) You would have to add 2 groups of 3 negative tiles. ? = +2 Therefore, (-6) ÷ (-3) = (+2)	
6. Students play the game Salute in groups of 3 to practice their integer multiplication concepts. 2 students are playing against each other to find their missing factor first. The 3rd player is the moderator. The moderator tells the 2 players to "salute" by holding a card up to their forehead. They show each other the cards but cannot see the card held to their head. The moderator mentally computes the product of the 2 cards and states the product aloud. Each player tries to figure out their card first. Whoever states their card correctly first, captures both cards for their win pile. If there is a tie, the moderator holds the cards until the next round is won. That winner also captures the tie pile. *(From Box Cars and One-Eyed Jacks)*	

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

Segment 5

Approximate Time Frame: 40-50 minutes	**Lesson Format:** ☐ Whole Group ☒ Small Group ☐ Independent ☐ Modeled ☒ Guided ☒ Collaborative ☐ Assessment	**Resources:** Modeling Multiplication and Division on Number Lines
Focus: Modeling integer multiplication and division with number lines, equations, and real-world situations.		**Modalities Represented:** ☐ Concrete/Manipulative ☒ Picture/Graph ☐ Table/Chart ☒ Symbolic ☒ Oral/Written Language ☒ Real-Life Situation

Math Practice Look For(s): **MP1**: Students explain and demonstrate rational number operations by using symbols, visuals, words, and real-life contexts. **MP2:** Students demonstrate quantitative reasoning by representing and solving real-world situations using visuals, numbers, and symbols. **MP3:** Students apply properties to support their arguments and constructively critique the reasoning of others while supporting their own position.	**Differentiation for Remediation:** **Differentiation for English Language Learners:** **Differentiation for Enrichment:**
Potential Pitfall(s):	**Independent Practice (Homework):**

Steps:	**Teacher Notes/Reflections:**
1. Post an empty number line on the board. Give the equation: 4 ● (-3) = n and have students work independently to model it on an empty number line.	
2. Have students share their representations with a colleague and discuss if they created the same model or different models. Select students to model their representations for the whole class.	**Teacher Notes/Reflections:**
3. Have them collaborate with their partner to describe a possible real-world situation that could be represented by either the equation or the number line diagrams.	**Teacher Notes/Reflections:**
4. Select students to share their situations with the whole group and allow students to ask clarifying questions or critique the examples that are provided.	**Teacher Notes/Reflections:**

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

5. Have students complete the Modeling Multiplication and Division on Number Line Diagrams page by working either with a partner or independently.	**Teacher Notes/Reflections:**

Segment 6

Approximate Time Frame:

50-60 minutes

Lesson Format:

- ☐ Whole Group
- ☒ Small Group
- ☐ Independent

- ☐ Modeled
- ☐ Guided
- ☒ Collaborative
- ☐ Assessment

Resources:

Modeling Multiplication and Division (from segment 5)

Solution and Critique template

Focus:

Solving problems involving multiplication and division of integers in real-world contexts.

Modalities Represented:

- ☐ Concrete/Manipulative
- ☒ Picture/Graph
- ☐ Table/Chart
- ☒ Symbolic
- ☒ Oral/Written Language
- ☒ Real-Life Situation

Math Practice Look For(s):

MP1: Students explain and demonstrate rational number operations by using symbols, visuals, words, and real-life contexts.

MP2: Students demonstrate quantitative reasoning by representing and solving real-world situations using visuals, numbers, and symbols.

MP3: Students apply properties to support their arguments and constructively critique the reasoning of others while supporting their own position.

Differentiation for Remediation:

Differentiation for English Language Learners:

Differentiation for Enrichment:

Potential Pitfall(s):

It is likely that students may confuse situations that are multiplicative in nature with those that are additive in nature. This is both a pitfall and an instructional discussion point. If students have confused these situations, select at least 1 of those additive situations to include in the 6 problems. This misconception will be addressed in the critique and debriefing portions of this segment.

Independent Practice (Homework):

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

Steps:	**Teacher Notes/Reflections:**
1. Select 6 contexts for word problems created by the students in segment 5 and change the numbers to create new problems. Put each problem in the Solution and Critique template and make copies for each student.	
2. Assign students to model and solve the new word problems in partners or triads.	**Teacher Notes/Reflections:**
3. After solving, have students analyze the contexts for representing multiplication and division situations involving both positive and negative numbers.	**Teacher Notes/Reflections:**
4. Facilitate a whole group discussion of both the solution processes and the contextual analysis for each word problem.	

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

Segment 7

Approximate Time Frame: 45-60 minutes	**Lesson Format:** ☒ Whole Group ☒ Small Group ☒ Independent ☐ Modeled ☒ Guided ☒ Collaborative ☒ Assessment	**Resources:** Word Problem Cards 2 color counters 9 x 18 Paper (1 per problem) Solving Integer Problems Formative Assessment
Focus: Solving problems with rational numbers and their four number operations in real-world contexts.		**Modalities Represented:** ☐ Concrete/Manipulative ☒ Picture/Graph ☐ Table/Chart ☒ Symbolic ☒ Oral/Written Language ☒ Real-Life Situation

Math Practice Look For(s): **MP1**: Students explain and demonstrate rational number operations by using symbols, visuals, words, and reallife contexts. **MP2:** Students demonstrate quantitative reasoning by representing and solving real world situations using visuals, numbers, and symbols. **MP3:** Students apply properties to support their arguments and constructively critique the reasoning of others while supporting their own position.	**Differentiation for Remediation:** **Differentiation for English Language Learners:** **Differentiation for Enrichment:**
Potential Pitfall(s):	**Independent Practice (Homework):**

Steps: 1. Students will be rotating with a partner or partners to 4 stations. Replicate enough problems so that each group rotates in their cycle to the 4 stations. *(For example, in a class of 24, 3 sets of cards will be needed.)* Post the word problem cards around the room. Assign work partners. Students solve the first problem they're assigned, representing their solution using either a diagram, manipulative, or an equation.	**Teacher Notes/Reflections:**
2. Students rotate to the next word problem and solve the problem using a different representation than the previous group—adding their representation to the one left behind by the previous group.	**Teacher Notes/Reflections:**

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

3. Students rotate to the next word problem and solve it using a different representation than either of the 2 previous groups—adding their representation to those left behind by the previous groups.	**Teacher Notes/Reflections:**
4. Students rotate to the next word problem and critique the work of the previous 3 groups who encountered that problem. They write about representations with which they agree and disagree . They can also reflect about representations that were unique, creative, or new to them.	
5. Bring students back to the whole group and discuss the various reflections from step 4.	
6. Give students the Solving Integer Problems Formative Assessment.	

Grade: 7 Lesson Title: Multiplying and Dividing Integers

Unit 3: Rational Number Operations *(Lesson 4 of 5)* **Time Frame:** 8-10 days

Essential Question:

- What is the relationship between properties of operations and types of numbers?

Segment 8

Approximate Time Frame: 40-50 minutes	**Lesson Format:** ☒ Whole Group ☐ Small Group ☐ Independent	**Resources:** Formative Assessment 7.NS.3
Focus: Assessment of student understanding and growth.	☐ Modeled ☐ Guided ☐ Collaborative ☒ Assessment	**Modalities Represented:** ☐ Concrete/Manipulative ☒ Picture/Graph ☐ Table/Chart ☒ Symbolic ☐ Oral/Written Language ☐ Real-Life Situation

Math Practice Look For(s):	**Differentiation for Remediation:** **Differentiation for English Language Learners:** **Differentiation for Enrichment:**
Potential Pitfall(s):	**Independent Practice (Homework):**

Steps: 1. Administer the Formative Assessment 7.NS.3.	**Teacher Notes/Reflections:**
2.	**Teacher Notes/Reflections:**
3.	**Teacher Notes/Reflections:**