

Connections to previous learning:

In Grade 6, students develop an understanding of ratio and proportion using ratio tables, tape diagrams, and double number lines.

Focus of this unit:

Students extend their understanding of ratios and develop understanding of proportionality to solve single and multi-step problems involving such real world contexts as percent of increase or decrease and scale drawing.

Connections to Subsequent Learning:

Students will use this understanding of proportionality to find scale factors between geometric figures and develop understandings of congruence and similarity. They will use ratio tables to study statistics and probability.

From the 6-7, Ratios and Proportional Relationships Progression Document, pp.10-11:

Multi-step problems Students extend their work to solving multi-step ratio and percent problems. 7.RP.3 Problems involving percent increase or percent decrease require careful attention to the referent whole. For example, consider the difference in these two percent decrease and percent increase problems:

Skateboard problem 1. After a 20% discount, the price of a SuperSick skateboard is \$140. What was the price before the discount?

Skateboard problem 2. A SuperSick skateboard costs \$140 now, but its price will go up by 20%. What will the new price be after the increase?

The solutions to these two problems are different because the 20% refers to different wholes or 100% amounts. In the first problem, the 20% is 20% of the larger pre-discount amount, whereas in the second problem, the 20% is 20% of the smaller pre-increase amount.

Notice that the distributive property is implicitly involved in working with

Using percentages in comparisons

There are 25% more seventh graders than sixth graders in the after-school club. If there are 135 sixth and seventh graders altogether in the after-school club, how many are sixth graders and how many are seventh graders?

100% of the sixth graders
Sixth graders: 25% of 6th graders, 25% of 6th graders, 25% of 6th graders, 25% of 6th graders
Seventh graders: same amount as 25% of the sixth graders

135 kids

9 parts → 135
1 part → 135 ÷ 9 = 15
4 parts → 4 × 15 = 60
5 parts → 5 × 15 = 75

60 sixth graders
75 seventh graders

"25% more seventh graders than sixth graders means that the number of extra seventh graders is the same as 25% of the sixth graders."

Skateboard problem 1

original 100% \$x
discounted 80% \$140

After a 20% discount, the price is 80% of the original price. So 80% of the original is \$140.

$x = \text{original price in dollars}$

percent	dollars
80% →	\$140
20% →	\$35
100% →	\$175

or add 80%+20%
"To find 20% I divided by 4. Then 80% plus 20% is 100%"

80% of the original price is \$140.

$$\frac{80}{100}x = 140$$

$$\frac{4}{5}x = 140$$

$$x = 140 \div \frac{4}{5} = 140 \cdot \frac{5}{4} = \frac{(2 \cdot 7 \cdot 2 \cdot 5) \cdot 5}{4} = 175$$

Before the discount, the price of the skateboard was \$175.

Skateboard problem 2

original 100% \$140
new, increased 120% \$x

After a 20% increase, the price is 120% of the original price. So the new price is 120% of \$140.

$x = \text{increased price in dollars}$

percent	dollars
100% →	\$140
20% →	\$28
100% →	\$168

or add 100%+20%
"To find 20% I divided by 5. Then 100% plus 20% is 120%"

The new, increased price is 120% of \$140.

$$x = \frac{120}{100} \cdot 140 = \frac{2 \cdot 6 \cdot 10}{2 \cdot 5 \cdot 10} \cdot 14 \cdot 2 \cdot 5 = 168$$

The new price after the increase is \$168.

percent decrease and increase. For example, in the first problem, if x is the original price of the skateboard (in dollars), then after the 20% discount, the new price is $x - 20\% x$. The distributive property shows that the new price is $80\% x$: $x - 20\% x = 100\% x - 20\% x = (100\% - 20\%) x = 80\% x$

Percentages can also be used in making comparisons between two quantities. Students must attend closely to the wording of such problems to determine what the whole or 100% amount a percentage refers to.

Connection to Geometry: One new context for proportions at Grade 7 is scale drawings. To compute unknown lengths from known lengths, students can set up proportions in tables or equations, or they can reason about how lengths compare multiplicatively. Students can use two kinds of multiplicative comparisons. They can apply a scale factor that relates lengths in two different figures, or they can consider the ratio of two lengths within one figure, find a multiplicative relationship between those lengths, and apply that relationship to the ratio of the corresponding lengths in the other figure. When working with areas, students should be aware that areas do not scale by the same factor that relates lengths. (Areas scale by the square of the scale factor that relates lengths, if area is measured in the unit of measurement derived from that used for length.)

Desired Outcomes

Standard(s):

Analyze proportional relationships and use them to solve real-world and mathematical problems.

- **7.RP.3** Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Draw, construct, and describe geometrical figures and describe the relationships between them.

- **7.G.1** Solve problems involving scale drawing of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing in a different scale.

Supporting Standards:

- **7.NS.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
 - a) Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers, interpret products of rational numbers by describing real-world contexts.
 - b) Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/1) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
 - c) Apply properties of operations as strategies to multiply and divide rational numbers.
 - d) Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational number.
- **7.EE.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that “increase by 5% is the same as multiply by 1.05.”*
- **7.EE.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $1/10$ of her salary an hour, or \$2.50 for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

Transfer:

Students will apply concepts and procedures for representing, interpreting and solving multi-step ratio and percent problems and problems involving scale drawings of geometric figures.

Ex: Sally bought a \$1400 certificate of deposit. At the end of 6 months, she received \$112 in simple interest. What annual rate of interest did the certificate pay?

Ex: Ms. Acton bought a few clothing items that added up to \$205.60. If the sales tax is 6% and Ms. Acton had a coupon for 20% off, what is the discount and total of her purchase?

Ex: A model plane is built to a scale of 1 in : 5ft. The length of the model is 18 in. What is the actual length of the plane?

Understandings: Students will understand that ...

- Rates, ratios, percentages and proportional relationships can be applied to problem solving situations such as interest, tax, discount, etc.
- Rates, ratios, percentages and proportional relationships can be applied to solve multi-step ratio and percent problems.
- Scale drawings can be applied to problem solving situations involving geometric figures.
- Geometrical figures can be used to reproduce a drawing at a different scale.

Essential Questions:

- How can I use proportional relationships to solve ratio and percent problems?
- How can I use scale drawings to compute actual lengths and area?
- How can I use geometric figures to reproduce a drawing at a different scale?

Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

- *1. Make sense of problems and persevere in solving them.** Students exhibit this standard when they represent and interpret proportional relationships to solve ratio and percent problems using visual models, proportions and other equations. They also make sense of proportional situations that involve scale drawings using diagrams and equations.
- *2. Reason abstractly and quantitatively.** Students will reason about the value of rational numbers in real-world contexts when representing and solving problems. They will apply proportional reasoning to scale drawings and determine if calculations are appropriate to the contexts.
- 3. Construct viable arguments and critique the reasoning of others.** Students will be expected to articulate their problem solving processes and explain the connection between the various representations (visual, tabular, algebraic, real-life) used to solve problems involving scale drawings and other proportional relationships.
- *4. Model with mathematics.** Students will use double number lines, tape diagrams and ratio charts to represent real-world situations involving proportional relationships.
- 5. Use appropriate tools strategically.** Students are expected to select appropriate measurement and construction tools when reproducing scale drawings.
- 6. Attend to precision.** Students will attend to the units of measurement when solving and representing problems with scale drawings. They will also attend to precise mathematical language when interpreting and communicating about problem solving with ratio and percent situations.
- 7. Look for and make use of structure.** Students look for patterns when solving proportional relationships and interpreting scale drawings.
- 8. Look for and express regularity in repeated reasoning.** Students use repeated reasoning when they replicate drawings at different scales.

Prerequisite Skills/Concepts:*Students should already be able to:*

- Understand ratio concepts and use ratio reasoning to solve problems. (6.RP.1-3)
- Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. (7.RP.1)
- Recognize and represent proportional relationships between quantities. (7.RP.2)

Advanced Skills/Concepts:*Some students may be ready to:*

- Understand the connections between proportional relationships to interpret unit rate as the slope of the graph. (8.EE.5)
- Proportional relationships can be applied to solve congruence and similarity applications. (8.G.2)

Knowledge: *Students will know...*

All standards for this unit go beyond the knowledge level.

Skills: *Students will be able to...*

- Solve multi-step ratio and percent problems. (7.RP.3)
- Solve problems involving simple interest and tax. (7.RP.3)
- Solve problems involving markups and markdowns, gratuities and commissions, and fees. (7.RP.3)
- Solve problems involving percent increase, percent decrease, and percent (margin of) error. (7.RP.3)
- Solve problems involving scale drawings of geometric figures. (7.G.1)
- Compute actual lengths and areas from a scale drawing. (7.G.1)
- Reproduce a scale drawing at a different scale. (7.G.1)

WIDA Standard: (English Language Learners)

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

English language learners will benefit from:

- hands-on and virtual manipulative experiences using geometric figures and fraction, decimal, percent equivalence tools.
- explicit vocabulary instruction with regard to ratio and percent situations.

Academic Vocabulary:**Critical Terms:**

Percent increase	Tax
Percent decrease	Gratuity
Percent error	Commission
Markups	Fee
Markdowns	Dimension
Discount	Simple interest
Scale	Proportion
Scale factor	

Supplemental Terms:

Area
Equivalent
Ratio

Assessment			
Pre-Assessments	Formative Assessments	Summative Assessments	Self-Assessments
1 Visual Models 11 Scale Drawing Pre-Assessment	5 On Sale! Activity 6 Percent Scavenger Hunt 7 Tip-Tax Activity 9 Percent Game 10 Solving Multi-Step Problems 12 Scale Drawing Project	13 New Flooring for the House 14 Unit Two Assessment	10 Solving Multi-Step Problems Self-Assessment Skeleton 12 Scale Drawing Project Self-Assessment Checklist
Sample Lesson Sequence			
1. 7.RP.3 Taxes, Fees, Commissions, Gratuities (Model Lesson) 2. 7.RP.3 Markups, Markdowns & Simple Interest 3. 7.RP.3 Percent of Increase, Decrease and Error 4. 7.RP.3 Solving multi-step ratio and proportion problems 5. 7.G.1 Scale drawing (Model Lesson)			