*Approximate time frame: 4 – 5 weeks*

**Connections to Previous Learning**:

Students in Grade 6 learn the concepts of ratio and unit rate as well as the precise mathematical language used to describe these relationships. They learn to solve problems using ratio and rate reasoning using a variety of tools such as tables, tape diagrams, double number lines and equations.

**Focus of this Unit**:

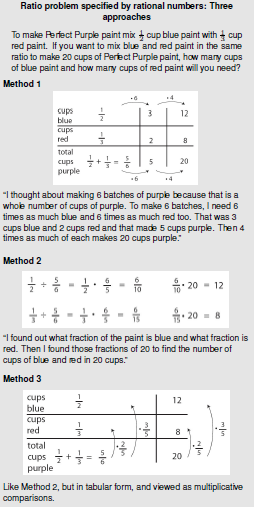
Students apply concepts of ratio and unit rate learned in grade 6 to fluently compute unit rates, represent proportional relationships between quantities, and compare and contrast proportional relationships in real world contexts.

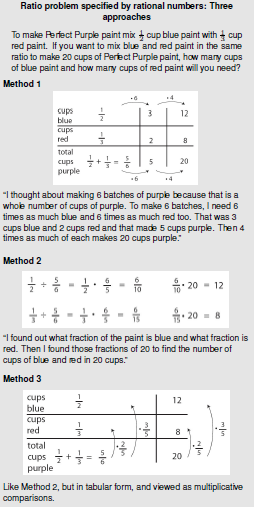
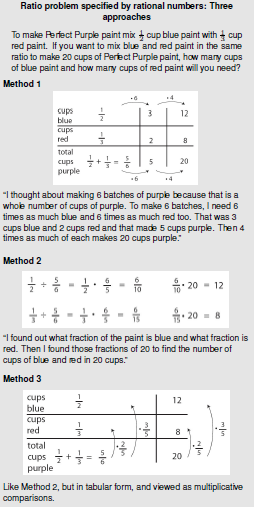
**Connections to Subsequent Learning:**

Students will apply their understanding of ratios and proportionality to situations involving multi-step ratio and percent problems as well as scale drawings.

*From the 6-7 Ratios and Proportional Reasoning Progression Document, pp. 8-10:*

In Grade 7, students extend their reasoning about ratios and proportional relationships in several ways. Students use ratios in cases that involve pairs of rational number entries, and they compute associated unit rates. They identify these unit rates in representations of proportional relationships. They work with equations in two variables to represent and analyze proportional relationships. They also solve multi-step ratio and percent problems, such as problems involving percent increase and decrease.

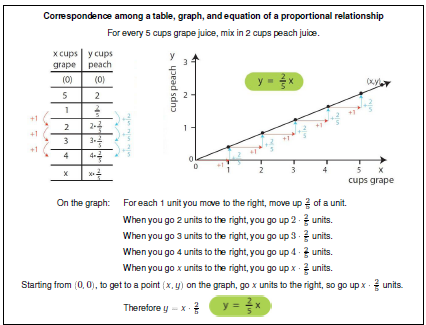
At this grade, students will also work with ratios specified by rational numbers, such as Students continue to use ratio tables, extending this use to finding unit rates.



**Recognizing proportional relationships** Students examine situations carefully, to determine if they describe a proportional relationship. For example, if Josh is 10 and Reina is 7, how old will Reina be when Josh is 20? We cannot solve this problem with the proportion 10/7 = 20/R because it is not the case that for every 10 years that Josh ages, Reina ages 7 years. Instead, when Josh has aged 10 another years, Reina will as well, and so she will be 17 when Josh is 20.

For example, if it takes 2 people 5 hours to paint a fence, how long will it take 4 people to paint a fence of the same size (assuming all the people work at the same steady rate)? We cannot solve this problem with the proportion 2/5 = 4/h because it is not the case that for every 2 people, 5 hours of work are needed to paint the fence. When more people work, it will take fewer hours. With twice as many people working, it will take half as long, so it will take only 2.5 hours for 4 people to paint a fence. Students must understand the structure of the problem, which includes looking for and understand the roles of “for every,” “for each,” and “per.”

Students recognize that graphs that are not lines through the origin and tables in which there is not a constant ratio in the entries do not represent proportional relationships. For example, consider circular patios that could be made with a range of diameters. For such patios, the area (and therefore the number of pavers it takes to make the patio) is not proportionally related to the diameter, although the circumference (and therefore the length of stone border it takes to encircle the patio) is proportionally related to the diameter. Note that in the case of the circumference, *C*, of a circle of diameter *D*, the constant of proportionality in *C* = pi *D* is the number *pi*, which is not a rational number.

**Equations for proportional relationships** As students work with proportional relationships, they write equations of the form *y* = *cx*, where *c* is a constant of proportionality, i.e., a unit rate. They see this unit rate as the amount of increase in *y* as *x* increases by 1 unit in a ratio table and they recognize the unit rate as the vertical increase in a “unit rate triangle” or “slope triangle” with horizontal side of length 1 for a graph of a proportional relationship.

Students connect their work with equations to their work with tables and diagrams. For example, if Seth runs 5 meters every 2 seconds, then how long will it take Seth to run 100 meters at that rate? The traditional method is to formulate an equation, 5/2 = 100/T, cross-multiply, and solve the resulting equation to solve the problem. If 5/2 and 100/T are viewed as unit rates obtained from the equivalent ratios 5 : 2 and 100 : T , then they must be equivalent fractions because equivalent ratios have the same unit rate. To see the rationale for cross-multiplying, note that when the fractions are given the common denominator 2 •T, then the numerators become 5 • T and 2 • 100 respectively. Once the denominators are equal, the fractions are equal exactly when their numerators are equal, so 5•Tmust equal 2 •100 for the unit rates to be equal. This is why we can solve the equation 5 •T = 2 •100 to find the amount of time it will take for Seth to run 100 meters.

A common error in setting up proportions is placing numbers in incorrect locations. This is especially easy to do when the order in which quantities are stated in the problem is switched within the problem statement. For example, the second of the following two problem statements is more difficult than the first because of the reversal.

“If a factory produces 5 cans of dog food for every 3 cans of cat food, then when the company produces 600 cans of dog food, how many cans of cat food will it produce?”

“If a factory produces 5 cans of dog food for every 3 cans of cat food, then how many cans of cat food will the company produce when it produces 600 cans of dog food?”

Such problems can be framed in terms of proportional relationships and the constant of proportionality or unit rate, which is obscured by the traditional method of setting up proportions. For example, if Seth runs 5 meters every 2 seconds, he runs at a rate of 2.5 meters per second, so distance d (in meters) and time t (in seconds) are related by d = 2:5t. If d=100 then t = 100/ 2:5 =40, so he takes 40 seconds to run 100 meters.

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| Desired Outcomes | |
| **Standard(s):**  **Analyze proportional relationships and use them to solve real-world and mathematical problems**   * 7.RP.1 Compute unit rates associates with rations of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4miles per hour, equivalently 2 miles per hour.* * 7.RP.2 Recognize and represent proportional relationships between quantities.   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.  b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.  c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*  d*.* Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. | |
| **Supporting Standards:**   * 7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.  1. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* 2. Understand *p + q* as the number located a distance │q│ from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. 3. Understand subtraction of rational numbers as adding additive inverse, *p- q = p + (-q).* Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. 4. Apply properties of operations as strategies to add and subtract rational numbers.  * 7. NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.   1. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers, interpret products of rational numbers by describing real-world contexts.   2. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then *–(p/q) = (-p)/q = p/(-q).* Interpret quotients of rational numbers by describing real-world contexts.   3. Apply properties of operations as strategies to multiply and divide rational numbers.   4. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. * 7. NS.3 Solve real-world and mathematical problems involving the four operations with rational number. * 7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a +0.05a = 1.05a means that “increase by 5% is the same as multiply by 1.05.”* | |
| **Transfer:**  Students will apply concepts and procedures for representing and analyzingratios, rates, and proportional relationships to solve real-world and mathematical problems.  Ex: Michael is running at a rate of 7 miles per hour, how many feet per minute is this?  Ex: Derek is 6.5 feet tall and casts a shadow that is 10 feet long. If a basketball hoop next to Derek is 10 feet tall, how long is the shadow casted by the hoop? | |
| **Understandings:** *Students will understand that …*   * Rates, ratios, and proportional relationships express how quantities change in relationship to each other. * Rates, ratios, and proportional relationships can be represented in multiple ways. * Rates, ratios, and proportional relationships can be applied to problem solving situations. | |
| **Essential Questions:**   * How do rates, ratios, and proportional relationships apply to our world? * When and why do I use proportional comparisons? * How does comparing quantities describe the relationship between them? * How do graphs illustrate proportional relationships? | |
| **Mathematical Practices: (Practices to be explicitly emphasized are indicated with an \*.)**  \*   1. **Make sense of problems and persevere in solving them.** Students make sense of ratio and unit rates in real-world contexts. They persevere by selecting and using appropriate representations for the given contexts.   \*   1. **Reason abstractly and quantitatively.** Students will reason about the value of the rational number in relation the models that are created to represent them. 2. **Construct viable arguments and critique the reasoning of others.**   \*   1. **Model with mathematics.**  Students create models using tape diagrams, double number lines, manipulatives, tables and graphs to represent real-world and mathematical situations involving ratios and proportions. For example, students will examine the relationships between slopes of lines and ratio tables in the context of given situations. 2. **Use appropriate tools strategically.**   \*   1. **Attend to precision.** Students attend to the ratio and rate language studied in grade 6 to represent and solve problems involving rates and ratios. 2. **Look for and make use of structure.** 3. **Look for and express regularity in repeated reasoning.** | |
| **Prerequisite Skills/Concepts:**  *Students should already be able to:*   * Understand ratio concepts and use ratio reasoning to solve problems. (6.RP 1-3) | **Advanced Skills/Concepts:**  *Some students may be ready to:*   * Analyze proportional relationships and use them to solve real-world and mathematical problems**.** (Compute unit rates. Recognize, represent and explain proportional relationships using tables, graphs, equations, diagrams and verbal descriptions. Use proportional relationships to solve multi-step ratio and percent problems.) *(7.RP.1-3)* * Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. *(7.NS.1-3)* * Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients. *(7.EE.1)* * Solve real-life and mathematical problems using numerical and algebraic expressions and equations. *(7.EE.3-4)* |
| **Knowledge**: *Students will know…*  All standards in this unit go beyond the knowledge level. | **Skills**: *Students will be able to …*   * Compute unit rates involving rational numbers, fractions, and complex fractions. (7.RP.1) * Compute ratios of length in like or different units. (7.RP.1) * Compute ratios of area and other measurements in like or different units. (7.RP.1) * Determine whether two quantities are in a proportional relationship by using a table and or graph. (7.RP.2) * Identify the constant of proportionality (unit rate) in tables, graphs, diagrams, and verbal descriptions. (7.RP.2) * Create and solve equations to represent proportional relationships. (7.RP.2) * Use words to describe the location of a point on a graph and its relationship to the origin. (7.RP.2) * Explain what a point on a graph of a proportional relationship means in terms of the situation. (how does the one quantity relate to the other) (7.RP.2) |

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| **WIDA Standard: (English Language Learners)**  English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.  English Language Learners benefit from:   * visual representations such as double number lines and bar models for representing and solving problems involving proportional relationships. * attention to the language of problems to determine ratio and rate representations appropriate to given contexts. | | | |
| **Academic Vocabulary:** | | | |
| **Critical Terms:**  Ratio  Rate  Proportion |  | **Supplemental Terms:**  Percent Proportion  Unit rate  Equivalency  Greatest Common Factor (GCF)  Least Common Multiple (LCM) |  |
| **Assessment** | | | |
| **Pre-Assessments** | **Formative Assessments** | **Summative Assessments** | **Self-Assessments** |
| Unit 1 Pretest  What is Unit Rate? | 7.RP.1 Formative Assessment  7.RP.2 Formative Assessment  Fraction Scavenger Hunt | Unit 1 Summative Assessment | Skeletons (7.RP.1, 7.RP.2, Unit) |
| **Sample Lesson Sequence** | | | |
| **1. 7.RP.1 - Unit rates and complex fractions (model lesson)**  2. 7.RP.2abc Recognizing and representing proportional relationships in equations, tables, and graphs  3. 7.RP.2cd Recognizing and representing proportional relationships in verbal descriptions, diagrams, and situations | | | |