Math 2 **N.CN.1, N.CN.2**

Assessment Title: Operations on Complex Numbers
Unit 1: Extending the Number System

Learning Targets:

- To know there is a complex number *I* such that $i^2 = -1$.
- To understand that every complex number has the form *a + bi* with *a* and *b* real.
- To perform arithmetic operations of addition, subtraction, and multiplication on complex numbers.

Part I: Introducing Imaginary Numbers

1. Earlier in this unit, we practiced solving equations of the form $x^2 = a$, for $a \geq 0$. Solve the following equations:
   a. $x^2 = 121$ b. $5x^2 = 200$

2. We will now explore situations where $a < 0$. How would we solve an equation like this?

$$x^2 = -36$$

3. The imaginary unit i is introduced when we are asked to take the square root of a negative number.
   a. If we let $i = \sqrt{-1}$, what is $i^2 =$ _____? Show your work and explain each step.

   b. Using the information above, what is $\sqrt{-36} = \sqrt{36} * \sqrt{-1}$ = _____.

   c. So the solutions to $x^2 = -36$ are $x = \pm\sqrt{-36}$ or _____.

d. As a check, note that $6i * 6i = 36i^2 = (36) * (-1) = -36$

4. Solve the following equations using your conclusions from above:

a. $x^2 = -100$

b. $x^2 = -25$

c. $x^2 = -49$

d. $x^2 = -169$

e. $x^2 = -18$

f. $x^2 = -96$

g. $2x^2 = -50$

h. $4x^2 - 4 = -68$

i. $-3x^2 - 27 = 162$

j. $5(x^2 + 20) = -300$

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Part II: Powers of i

Using the information about the imaginary unit i you have learned above, calculate the following powers of *i*. The first two problems, $i^{\,1}\ and\ i^2$, have been completed for you.

$\boldsymbol{i^1 = i}$

$\boldsymbol{i^2 = -1}$

$i^3 =$ _____

$i^4 =$ _____

$i^5 =$ _____

$i^6 =$ _____

$i^7 =$ _____

$i^8 =$ _____

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$i^{100} =$ _____

$i^{101} =$ _____

Describe what patterns you recognize from completing the problems above.

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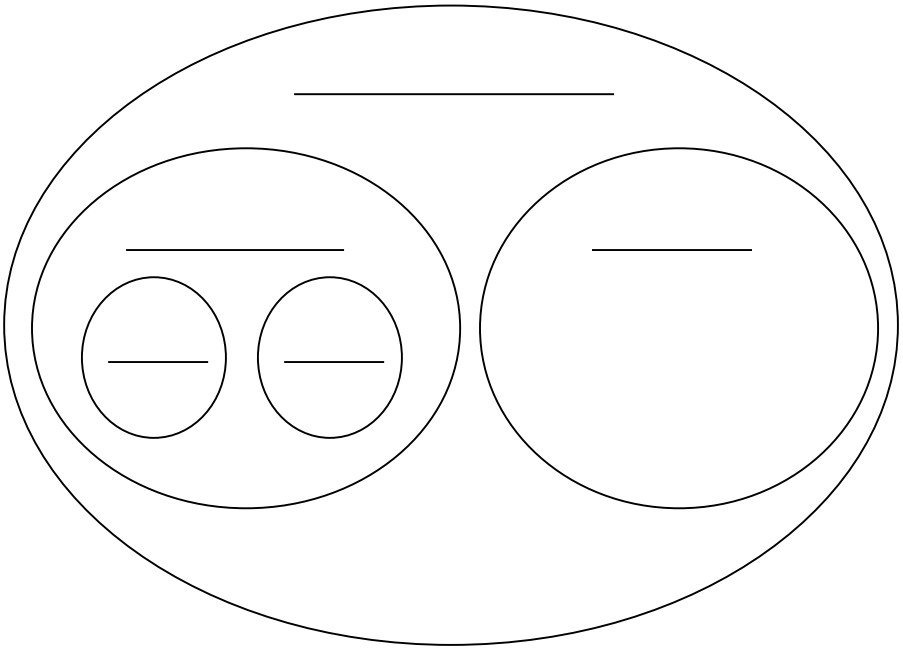
Part III: Complex Numbers and Operations

Imaginary numbers are part of a broader set of numbers called *complex numbers*. Using the word bank, fill in the diagram below, placing each type of number in its appropriate place. Give two examples of each type of number in each category.



Word Bank

Real Numbers
Imaginary Numbers
Complex Numbers
Rational
Irrational

Complex numbers are written in the form:

$$a + bi$$

where a is a(n) _____ number, and b is a(n) _____ number.

If b is zero, what type of number is represented? _____

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We can use addition and subtraction on complex numbers by combining like terms. Complete each problem below by writing an equivalent expression for each.

1. $(2 + 4i) + (5 - 7i)$
2. $(4 - 8i) - (3 - 6i)$
3. $3(-2 + 5i) - (1 - 7i)$
4. $\left(7 + \sqrt{-81}\right) + 17i$

We can also use multiplication on complex numbers. *Remember:* $i^2 = -1$. Complete each problem by writing an equivalent expression for each.

1. $(-6i)(-6i)$
2. $(-5i)(3i)$
3. $\sqrt{-6} * \sqrt{-15} * \sqrt{-80}$
4. $\left[\left(\frac{1}{2}i\right)^2\right] * (-2i)^2$
5. $-9i(4 - 3i)$

6. $(4 - 6i)(7 + i)$

7. $(4 - 6i)(6 - 6i)$

8. $(-2i + 7)(-2i - 7)$

9. $(5 + 3i)(5 - 3i)$