Assessment Title: The Tortoise, The Hare, and the Aardvark

Unit 5: Modeling and Comparing Functions

Learning Targets:

- Quadratic functions have key features that can be represented on a graph and can be interpreted to provide information to describe relationships of two quantities. These graphs can be compared to linear and exponential functions to model a situation.
- The meaning of average rate of change of a quadratic model is interpreted based upon the context.
- Equations are affected by transformations of a graph and vice versa.

In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race. The distance from the starting line of the hare is given by the function:

$d = t^2$ (d in meters and t in seconds)

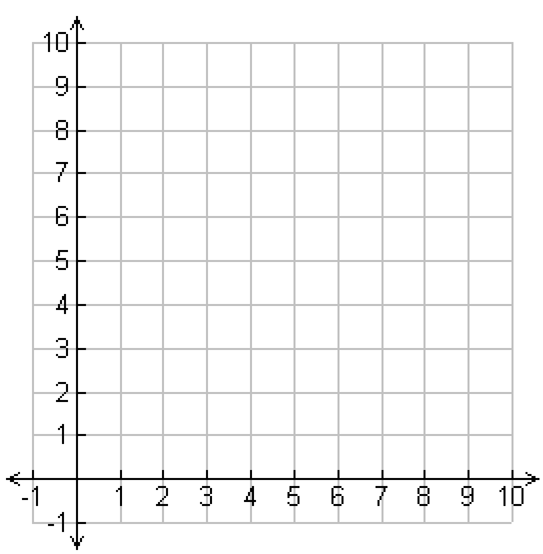
Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

$d = 2^t$ (d in meters and t in seconds)

A little known part of the story is that when the race was about to begin an aardvark came strolling over and decided that he thought he could beat both of them with his consistent trot that can be described by the function:

$d = 2t$ (d in meters and t in seconds)

1. Graph all three functions on the same grid to help see how the race is going to go.



2. Identify the key features of each function of the graph.

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3. Fill out the table.

X	0	1	2	3	4	5	6	7	8
$d = 2^t$									
$d = t^2$									
$d = 2t$									

4. At what time did the aardvark catch up to tortoise?

5. At what time does the hare catch up to the tortoise?

6. If the race course is very long, who wins: the tortoise, the hare, or the aardvark? Why?

7. Is there a point in time when all three are tied? If so, when is this? If not, when are they closest?

8. Is there a point when there is just a 2 way tie? If so, when is this?

9. If the race course were 3 meters long who wins, the tortoise, the hare, or the aardvark? Why?

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10. If the race course were 15 meters long who wins, the tortoise, the hare, or the aardvark? Why?

11. Use the properties $d = 2^t, d = t^2, and\ d = 2t$ to explain the speeds of the tortoise and the hare in the following time intervals:

Interval	Tortoise $d = 2^t$	Hare $d = t^2$	Aardvark $d = 2t$
$0 \le x < 1$			
$1 \le x < 2$			
$2 \le x < 4$			
$x \ge 4$			

12. What would happen to the equation $d = 2^t$ if the hare did not give a 1 meter head start to the tortoise? How does this change any of your previous conclusions?

13. What would happen to the equation $d = 2t$ if the aardvark got the 1 meter head start instead of the tortoise? How does this change any of your previous conclusions?

14. What happens to the equations $d = 2^t, d = t^2, and\ d = 2t$ if the Hare gave both competitors a 1 second head start instead of 1 meter? How does this change any of your previous conclusions?

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In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race. The distance from the starting line of the hare is given by the function:

$d = t^2$ (d in meters and t in seconds)

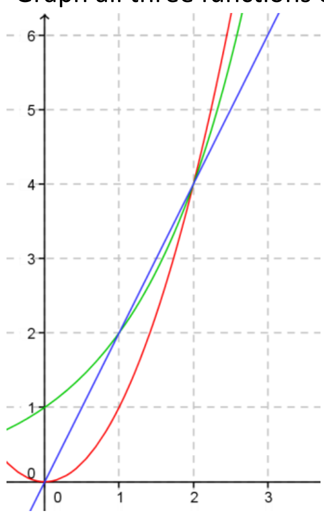
Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

$d = 2^t$ (d in meters and t in seconds)

A little known part of the story is that when the race was about to begin an aardvark came strolling over and decided that he thought he could beat both of them with his consistent trot that can be described by the function:

$d = 2t$ (d in meters and t in seconds)

1. Graph all three functions on the same grid to help see how the race is going to go.



2. Identify the key features of each function of the graph.

Hare: $d = t^2$: t – intercept: (0, 0); d – intercept: (0, 0)

Tortoise: $d = 2^t$: t – intercept: n/a; d – intercept: (0, 1)

Aardvark: $d = 2t$: t – intercept: (0, 0); d – intercept: (0, 0)

3. Fill out the table.

X	***0***	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***
$d = 2^t$	1	2	4	8	16	32	64	128	256
$d = t^2$	0	1	4	9	16	25	36	49	64
$d = 2t$	0	2	4	6	8	10	12	14	16

4. At what time did the aardvark catch up to tortoise? 1 second

5. At what time does the hare catch up to the tortoise? 2 seconds

6. If the race course is very long, who wins: the tortoise, the hare, or the aardvark? Why?

   If the race exceeds 16 meters, the tortoise will win the race. The increasing exponential function will eventually always exceed a quadratic function.

7. Is there a point in time when all three are tied? If so, when is this? If not, when are they closest? Yes, at 2 seconds. They will all be 4 meters from the starting line.

8. Is there a point when there is just a 2 way tie? If so, when is this? A two-way tie occurs between the tortoise and aardvark at 1 second. They are both 2 meters from the starting line.

   Another two-way tie occurs between the tortoise and the hare at 4 seconds. They are both 16 meters from the starting line.

9. If the race course were 3 meters long who wins, the tortoise, the hare, or the aardvark? Why?

   The hare would win with a distance of 9 meters.

10. If the race course were 15 meters long who wins, the tortoise, the hare, or the aardvark? Why?

    The hare would win the race if it were 15 meters long. It would take the hare about 3.87 seconds, while the tortoise would follow at around 3.91 seconds, and the aardvark 7.5 seconds.

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11. Use the properties $d = 2^t, d = t^2, and\ d = 2t$ to explain the speeds of the tortoise and the hare in the following time intervals:

Interval	Tortoise $d = 2^t$	Hare $d = t^2$	Aardvark $d = 2t$
$0 \leq x < 1$	1 meter/second	1 meter/second	2 meters/second
$1 \leq x < 2$	2 meters/second	3 meters/second	2 meters/second
$2 \leq x < 4$	6 meters/second	6 meters/second	2 meters/second
$x \geq 4$	Answers vary	Answers vary	2 meters/second

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12. What would happen to the equation $d = 2^t$ if the hare did not give a 1 meter head start to the tortoise? How does this change any of your previous conclusions?

    The equation for the tortoise would change to d = $2^t - 1$. It would take a longer for the tortoise to pass both the aardvark and the hare.

13. What would happen to the equation $d = 2t$ if the aardvark got the 1 meter head start instead of the tortoise? How does this change any of your previous conclusions?

    The equation for the aardvark would change to d = 2t + 1.

14. What happens to the equations $d = 2^t, d = t^2, and\ d = 2t$ if the Hare gave both competitors a 1 second head start instead of 1 meter? How does this change any of your previous conclusions?

The equation for the hare would be d = $(x - 1)^2$ for times greater than 0 seconds. Instead of the hare catching up with the aardvark and tortoise at 2 seconds, he would not catch up to/pass the aardvark until after 3 seconds. He would never catch the tortoise.

X	***0***	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***
$d = 2^t$	1	2	4	8	16	32	64	128	256
$d = t^2$	0	1	4	9	16	25	36	49	64
$d = (t-1)^2$	0	0	1	4	9	16	25	36	49
$d = 2t$	0	2	4	6	8	10	12	14	16