*Approximate Time Frame: 5 weeks*

## Connections to Previous Learning:

Students in Grade 7 learn to differentiate between terminating and repeating decimals. In Grade 8, students realize that terminating decimals are repeating decimals that repeat the digit zero. They use this concept to identify irrational numbers as decimals that do not repeat a pattern. They learn to use rational approximations of irrational numbers to represent the value of irrational numbers on a number line. Students in Grades 6 and 7 have learned to use expressions, equations and inequalities to represent problem solving situations. Students in Grade 8 will expand upon those skills to include work with very large and very small numbers involving the use of integer exponents.

## Focus of this Unit:

Beginning with familiar number sense topics helps students transition into the Grade 8 content. Turning decimal expansions into fractions and deepening understanding of the meaning of decimal expansions sets a firm foundation for understanding irrational numbers. Students will learn that the square roots of perfect squares are rational numbers and that the square roots of non-perfect squares, such as $\sqrt{2}$ or $\sqrt{7}$ are examples of irrational numbers. Students will understand the value of square roots and cube roots and use this understanding to solve equations involving perfect squares and cubes. Further work with exponents, including scientific notation, naturally flow from the understanding of squares and cubes.

## Connections to Subsequent Learning:

Solving equations of the form $x^2 = p$ reminds students about inverse operations which they will need to explore the topics of solving linear and proportional equations later in the year. Exponents and roots also connect closely to work with the Pythagorean Theorem and volume of rounded objects later in Grade 8.

*From the 6-8, Expressions and Equations Progression Document, p. 11:*

**Work with radicals and integer exponents**

In Grade 8 students add the properties of integer exponents to their repertoire of rules for transforming expressions. Students have been denoting whole number powers of 10 with exponential notation since Grade 5, and they have seen the pattern in the number of zeros when powers of 10 are multiplied. They express $10^a 10^b = 10^{a+b}$ for whole numbers $a$ and $b$. Requiring this rule to hold when $a$ and $b$ are integers leads to the definition of the meaning of powers with 0 and negative exponents. For example, we define $10^0 = 1$ because we want $10^a 10^0 = 10^{a+0} = 10^a$, so $10^0$ must equal 1. Students extend these rules to other bases, and learn other properties of exponents.

**Properties of Integer Exponents**

For any nonzero rational numbers $a$ and $b$ and integers $n$ and $m$:

1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$
4. $a^0 = 1$
5. $a^{-n} = 1/a^n$

Notice that students do not learn the properties of rational exponents until high school. However, they prepare in Grade 8 by starting to work systematically with the square root and cube root symbols, writing, for example, $\sqrt{64} = \sqrt{8^2} = 8$ and $(\sqrt[3]{5})^3 = 5$. Since $\sqrt{p}$ is defined to mean the positive solution to the equation $x^2 = p$ (when it exists), it is not correct to say (as is common) that $\sqrt{64} = \pm 8$. On the other hand, in describing the solutions to $x^2 = 64$, students can write $x = \pm\sqrt{64} = \pm 8$. Students in grade 8 are not in a position to prove that these are the only solutions, but rather use informal methods such as guess and check.

Students gain experience with the properties of exponents by working with estimates of very large and very small quantities. For example, they examine the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$, and determine that the world population is more than 20 times larger. They express and perform calculations with very large numbers using scientific notation. For example, given that we breathe about 6 liters of air per minute, they estimate that there are $60 \times 24 = 6 \times 2.4 \times 10^2 \approx 1.5 \times 10^3$ minutes in a day, and that we therefore breathe about $6 \times 1.5 \times 10^3 \approx 10^4$ liters in a day. In a lifetime of 75 years there are about $365 \times 75 \approx 3 \times 10^4$ days, and so we breathe about $3 \times 10^4 \times 10^4 = 3 \times 10^8$ liters of air in a lifetime.

## Desired Outcomes

### Standard(s):

**Know that there are numbers that are not rational, and approximate them by rational numbers.**

- 8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
- 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^2$). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between $1$ and $2$, then between $1.4$ and $1.5$, and explain how to continue on to get better approximations.*

**Work with radicals and integer exponents.**

- 8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$.
- 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
- 8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$, and determine that the world population is more than $20$ times larger.*
- 8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

## Transfer:

Students will apply concepts and procedures involving irrational numbers, radicals, integer exponents, and scientific notation to represent, interpret and solve problems for real-world and mathematical situations.

Ex: A pet fish needs $1000\ in^3$ of open water. If you want your fish tank to be a cube, what should the side length be?

Ex: The Andromeda galaxy is approximately $1.5 \times 10^{19}$ miles. The distance from the sun to Earth is approximately $9.3 \times 10^{7}$ miles. Using appropriate technology, about how many times further away is the Andromeda galaxy than the sun? Using the output from the technology, demonstrate your ability to interpret this output by converting the output into standard form.

## Understandings: *Students will understand that …*

- Every number has a decimal expansion.
- The value of any real number can be represented in relation to other real numbers such as with decimals converted to fractions, scientific notation and numbers written with exponents ($8 = 2^3$).
- Properties of operations with whole and rational numbers also apply to all real numbers.

## Essential Questions:

- Why are quantities represented in multiple ways?
- How is the universal nature of properties applied to real numbers?

## Mathematical Practices: *(Practices to be explicitly emphasized are indicated with an *.)*

1. **Make Sense of Problems and Persevere in Solving Them**.

*2. **Reason Abstractly and Quantitatively.** Students have the opportunity to reason abstractly by constructing the integer exponent operation rules. From the definition of exponents, they should discover a pattern that will hold true for a power times a power, a power divided by a power, and a power to a power. By expressing these rules as generalized statements like $x^a x^b = x^{a+b}$, students can express their quantitative reasoning in an abstract manner.

3. **Construct Viable Arguments and Critique the Reasoning of Others.**
4. **Model with Mathematics.**
5. **Use Appropriate Tools Strategically.**

*6. **Attend to Precision.** Through exploration of scientific notation, students should see that precision is relative to need. For example, the average distance to the sun from Earth is listed as $92{,}956{,}050$ miles on the NASA website, but we would typically see it written as approximately $9.3 \times 10^7$ miles. In the case of general knowledge, it is sufficient precision to say that we are about 93 million miles from the sun, but for specific missions to or near the sun, more precision would be needed. For example, also listed on NASA's website is the distance in km which they gave both as $149{,}598{,}262$ km and as $1.4959826 \times 10^8$ km. Notice that for a NASA mission, the greater level of precision would be needed as expressed in their scientific notation calculation rather than rounding it to $1.5 \times 10^8$ km.

*7. **Look for and Make Use of Structure.** Scientific notation also offers a chance for students to make use of the structure of numbers. When performing addition with numbers given in scientific notation, students begin to develop the idea of a term, like a monomial term in a polynomial, because they can view the structure of a number in scientific notation as a single entity complete in itself. When performing multiplication with numbers given in scientific notation, students take advantage of the structure by multiplying the decimal parts and powers of ten separately. Students should see that varying structures can be used based on the need of the situation.

8. **Look for and Express Regularity in Repeated Reasoning.**

| **Prerequisite Skills/Concepts:** *Students should already be able to…* | **Advanced Skills/Concepts:** *Some students may be ready to…* |
|---|---|
| • Perform operations with rational numbers including negative rational numbers. (7.NS)<br>• Rewrite expressions in different forms. (7.EE.2) | • Identify real and complex numbers through the introduction of $i = \sqrt{-1}$.<br>• Reduce irrational numbers to simplest radical form. ($\sqrt{24} = 2\sqrt{6}$).<br>• Rationalizing fractions with a square root in the denominator.<br>• Multiply and divide monomials. ($(2x^{-3}y^5z)(3x^5y^{-3})$ *or* $(2x^{-3}y^5z)/(3x^5y^{-3})$). |
| **Knowledge:** *Students will know…* | **Skills:** *Students will be able to…* |
| • Decimals that "terminate" actually repeat the digit zero. *($2.5 = 2.500000$ ….)* (8.NS.1)<br>• Numbers that repeat in their decimal form are called rational. (8.NS.1)<br>• Numbers that do not repeat in their decimal form are called irrational. (8.NS.1)<br>• The number $\sqrt{2}$ is irrational. (8.EE.2)<br>• The square root of the area of a square represents the side length of the square. (8.EE.2)<br>• Exponent operation properties. (8.EE.1) | • Distinguish between rational and irrational numbers. (8.NS.1)<br>• Convert a decimal expansion which repeats eventually into a rational number. (8.NS.1)<br>• Convert a fraction into a repeating decimal. (8.NS.1)<br>• Find rational approximations of irrational numbers. (8.NS.2)<br>• Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions.(8.NS.2)<br>• Evaluate square roots of small perfect squares and cube roots of small perfect cubes. (8.EE.2)<br>• Use square root and cube root symbols to solve and represent solutions of equations. (8.EE.2)<br>• Apply the properties of integer exponents to generate equivalent numerical expressions. (8.EE.1)<br>• Estimate very large or very small quantities using a single digit times a power of ten. (8.EE.3)<br>• Express how much larger one number expressed as a single digit times a power of ten is than another in the context of the situation. (8.EE.3)<br>• Express numbers in scientific notation. (8.EE.4)<br>• Perform operations with numbers expressed in scientific notation and a mix of scientific notation and decimal notation. (8.EE.4)<br>• Choose appropriate units of measurements for a given number in scientific notation. (8.EE.4)<br>• Interpret scientific notation that has been generated by technology. (8.EE.4) |

**WIDA Standard: (English Language Learners)**

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
English language learners benefit from:

- explicit vocabulary instruction with regard to the critical terminology.
- the use of visuals and concrete manipulatives (such as color tiles and unit cubes for working with squares, cubes, square roots and cube roots).

| Academic Vocabulary: | |
|---|---|
| **Critical Terms:**<br>Exponent<br>Scientific notation<br>Radical<br>Irrational number<br>Rational number<br>Square root<br>Cube root<br>Perfect cube<br>Perfect square | **Supplemental Terms:**<br>Repetend<br>Equation<br>Expression<br>Variable<br>Property<br>Unknown<br>Solution<br>Integer<br>Inverse operations |

## Assessment

| Pre-Assessments | Formative Assessments | Summative Assessments | Self-Assessments |
|---|---|---|---|
| Prior Knowledge Pre-Test<br>Real Numbers Pre-Test<br>Exponents Pre-Test<br>Scientific Notation Test | Irrational Estimation Dice Game (8.NS.2)<br>Irrational Estimation Card Game (8.NS.2)<br>Number Line Code (8.NS.2)<br>Scavenger Hunt (8.NS.2) | Real Numbers Post-Test<br>Exponents Post-Test<br>Scientific Notation Post-Test<br>Star Trek Project | Rational and Irrational Identification (8.NS.1) |

## Sample Lesson Sequence

1. **8.NS.1 and 8.NS.2 – Real Numbers (model lesson)**
2. 8.EE.1 and 8.EE.2 – Exponents
3. 8.EE.3 and 8.EE.4 – Scientific Notation