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# **MATH for Manufacturing**

Student Workbook

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by Ray Prendergast

Version 1.43



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by Ray Prendergast

December 2013

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**Ray Prendergast**  
**Dean, College To Careers**  
**Richard J. Daley College, The Center of Excellence**  
**for the Manufacturing Technology Programs**  
**of the City Colleges of Chicago**  
**7500 S. Pulaski Road**  
**Chicago, IL 60652**  
**773-838-7786**  
**rprendergast@ccc.edu**

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# Acknowledgements

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Please note that the content of Unit 6 was developed by Guy Loudon for his drill press classes. Guy is currently the Executive Director of the Jane Addams Resource Corporation.

The following corporate partners provided access for photographs. In addition, many of the worksheet math problems are derived from their prints and manufacturing processes.

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7450 N. McCormick Blvd.  
Skokie, IL 60076-8103  
847-674-7700

[www.federalmogul.com](http://www.federalmogul.com)

Formerly Fel-Pro, the sealing systems plant in Skokie, IL is one of the leading producers of gaskets for the automotive industry. Federal Mogul is also an employment partner in the Chicago bridge program.

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Oakton Community College  
1600 E. Golf Road  
Des Plaines, IL 60016  
847-635-1932

[www.oakton.edu/cont\\_ed/bi/b\\_i.htm](http://www.oakton.edu/cont_ed/bi/b_i.htm)

Oakton's Business Institute is the training service provider for Federal Mogul Sealing Systems and MPC Products. Many of the worksheets in this book were utilized in shop math, print reading and SPC classes conducted by Oakton's Business Institute.

Jane Addams Resource Corporation  
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Chicago, IL 60640  
773-728-9769

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JARC, as it's known in Chicago, is a community-based economic development agency with a focus on the manufacturing sector. JARC provides training in CNC machining, welding, stamping, and forklift operation. Many of the worksheets in this book were used in JARC's metalworking skills program.

## About the Author

Ray Prendergast is the Dean of College to Careers for Richard J. Daley College, one of the City Colleges of Chicago. Daley College is the Center of Excellence for Advanced Manufacturing for the City Colleges and its Manufacturing Technology program includes an AAS degree and six college certificates in CNC Machining, Welding, Factory Automation, and Quality Assurance.

Ray has over 18 years of experience in training people for manufacturing jobs and over 17 years working in those manufacturing jobs including machining, precision sheet metal fabrication, rigging, stamping, and quality assurance.

Ray is currently revising this workbook to better serve Illinois community college programs "bridging" adult education students into manufacturing programs.



# Introduction

Today's business leaders are looking for flexible employees who can adapt to a work environment that is becoming more complex. These employers typically ask for workers who are good critical thinkers, excellent problem solvers, and can communicate clearly and concisely. They encourage their workers to take special training courses to stay current in their chosen fields of specialization.

This Workbook is designed to address the functional needs of adult learners in precision metalworking training programs. The core skill of precision metalworking is math. For many trainees fractions, four-place decimals, square roots and trig functions are the stuff of bitter school memories. This workbook is an attempt to replace such bitterness with confidence.

The worksheets in this book are intended to stimulate the interest of the participant and provide relevant points for discussion and team interaction. They will help you develop and practice mathematics skills you will use on the job. Each Math for Manufacturing unit can be used in independent study or in group instruction.

The worksheets should be particularly helpful to math teachers who are not fluent with manufacturing processes. Teachers can use these worksheets as a basis for developing additional practical math exercises that relate to industry practices.

Please note that this is not a complete textbook. It does not focus on basic math operations, but on their application in industry. Thus, other texts and instructional materials should be provided for a complete shop math course.

# To the learner

Welcome to Math for Manufacturing! This Workbook is designed to help you develop the skills needed to work with decimals, fractions, and precision measurements, and to apply these skills to real on-the-job problems.

The Workbook contains ten units. Each unit consists of several lessons that will help you master the given subject. You will use listening, speaking, reading, and writing skills in each lesson.

New vocabulary words are defined in the glossary in the back of the Workbook. An informative bibliography will help you delve further into various areas of concentration.

Note that many of the math problems in the worksheets are taken from blueprints. It is assumed that you will be learning to read prints at the same time as math. However, if you are not learning to read prints you may need a little help to understand the drawings. Ask your teachers or look for help in the print reading books listed in the bibliography.

Each unit of Mathematics for Manufacturing contains three sub-sections:

- MEET IT:           Get an overview of the concepts to be mastered
- LEARN IT:        Work to improve skills related to concepts
- USE IT:           Apply skills to real job situations

Talk with your instructor if you need extra help on any of the lessons. When answering questions and solving problems, make sure to check your answers carefully. When you have completed this Workbook, you will have learned and actually used many new mathematical skills that will help you on the job.

Good luck!

# Unit 1: Decimals

## MEET IT

The most essential factor in making anything is measurement. Measurements allow us to describe objects precisely and make them accurately. To a precision metalworker, the languages of measurement are as important as the spoken word.

Decimals are the language we use for precision measurement. To do quality precision metalwork, you have to work in the language of decimals. When you understand this language, you will be able to read dimensions exactly, measure physical objects accurately, and do basic math with the numbers you get correctly.

Every day machinists in the field of precision metalworking use rulers, micrometers, calipers, calculators, and computers to work in the language of measurement.



This electronic caliper is measuring the inner diameter of an aerospace component. To do this the inspector must work with numbers with 3 or 4 place decimals

## LEARN IT

In this unit you will learn to:

- Use the rules for decimal addition, subtraction, multiplication, and division
- Use a scientific calculator to do repetitive calculations with decimals
- Perform multi-step caliper math problems with two- and three-place decimals
- Perform addition, subtraction, multiplication, and division with four-place decimals

## USE IT

### Two-Place Decimals

- calculate upper and lower limits of dimensions
- caliper math with metric dimensions

### Three-Place Decimals

- caliper math with inch dimensions

### Four-Place Decimals

- leaving enough stock for grinding operations



Bertha is using a micrometer to measure the material thickness of an automotive gasket to within .001”.

## LEARN IT

In manufacturing we deal with decimal numbers more than fractions or whole numbers. Most people are familiar with 2-place decimals as dollars and cents. On metric prints the dimensions are given in millimeters and the numbers are one- or two-place decimals. Prints and parts for the automotive industry are usually metric.

The dimensions on a print are targets that you aim for. The target is not an exact number but a range and the measure of each feature must fall within that range.

Every dimension that must be measured has a tolerance. This tolerance is an amount that you add and subtract to the print dimension. For example:

$$88.4 \pm 0.25$$

The “nominal” size is 88.4, that’s what you aim for. The tolerance is “plus or minus .25” which means that this feature on a real part must measure somewhere between:

$$88.15 \text{ (= } 88.4 - .25\text{), the lower limit}$$

and

$$88.65 \text{ (= } 88.4 + .25\text{), the upper limit}$$

The part can measure right on the limit and that’s OK. If the part measures 81.14, or 88.66, then it’s bad and there is no argument about it.

When working from a print in a job shop, a sheet metal fabricator or a machinist will have to calculate the limits for every dimension. It is good practice to go through the print and write down the upper and lower limits next to each dimension.

Use the worksheet on the next page to practice calculating dimension limits from standard tolerances.

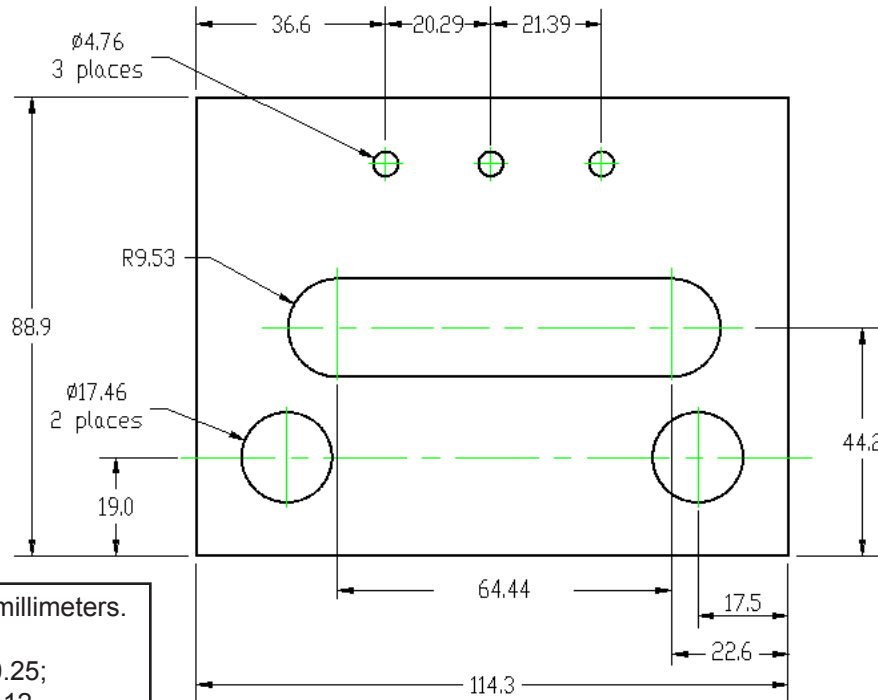
For this exercise you will use two different tolerances. Note that which tolerance you use depends on the number of decimal places in the dimension.

For dimensions that have one decimal place use  $\pm 0.25$  as the tolerance.  
If the dimension is a 2-place decimal, use  $\pm 0.12$

The first two problems are already done for you as examples.

**Print reading tip:** The symbol  $\varnothing$  means “diameter” which is the distance across a hole, while the symbol  $R$  means “radius,” or the distance from the center of circle or an arc.

### Worksheet 1-1: Metric Tolerances, 1- and 2-Place Decimals



Dimensions are in millimeters.  
Tolerances are:  
1-place decimal  $\pm 0.25$ ;  
2-place decimal  $\pm 0.12$

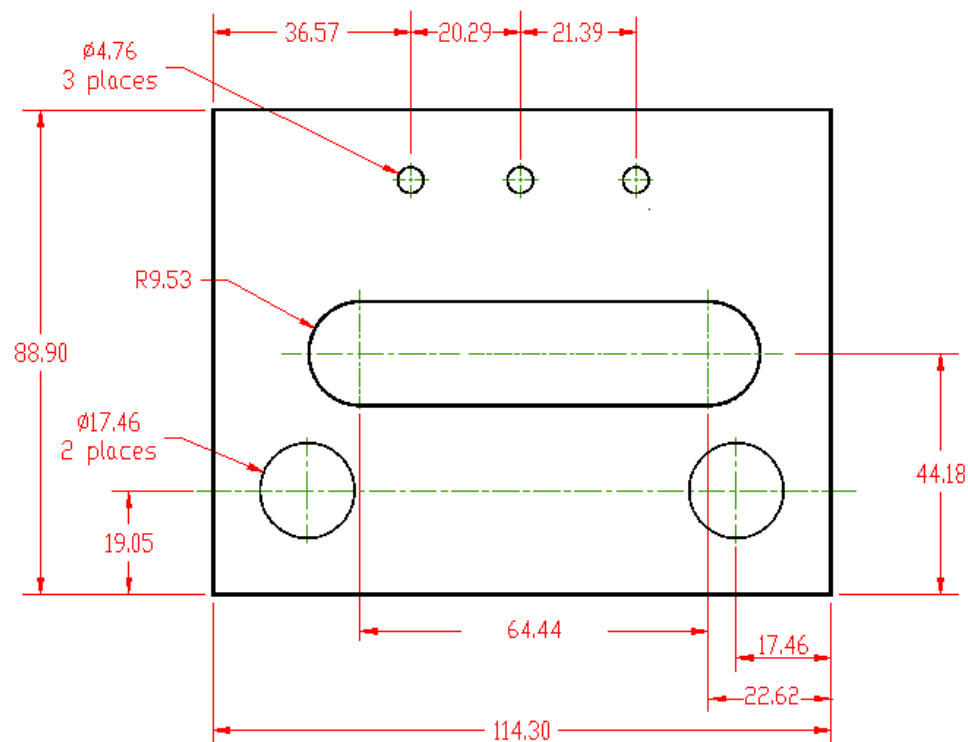
- |          |  |           |                              |
|----------|--|-----------|------------------------------|
| 1) 4.76  | Upper limit: $4.76 + .12 = 4.88$<br>Lower limit: $4.76 - .12 = 4.64$   | 7) 17.46  | Upper: _____<br>Lower: _____ |
| 2) 36.6  | Upper limit: $36.6 + .25 = 36.85$<br>Lower limit: $36.6 - .25 = 36.35$ | 8) 19.0   | Upper: _____<br>Lower: _____ |
| 3) 20.29 | Upper: _____<br>Lower: _____   | 9) 114.3  | Upper: _____<br>Lower: _____ |
| 4) 21.39 | Upper: _____<br>Lower: _____   | 10) 64.44 | Upper: _____<br>Lower: _____ |
| 5) 9.53  | Upper: _____<br>Lower: _____   | 11) 17.5  | Upper: _____<br>Lower: _____ |
| 6) 88.9  | Upper: _____<br>Lower: _____   | 12) 22.6  | Upper: _____<br>Lower: _____ |
|          |  | 13) 44.2  | Upper: _____<br>Lower: _____ |

## Worksheet 1-2: Caliper Math with 2-Place Decimals

The blueprint gives you the requirements for a part. It gives you the size and location of all the features on the part. Yet, you can't always measure the dimensions specified on the print. Sometimes, you have to calculate a dimension from measurements of other features.

For example, when you check holes drilled into a plate, you have to verify both the size and location of the hole.

You can measure the hole size directly with the inside jaws of your dial calipers and compare this with the print dimension for the hole diameter. However, the location of a hole is always given from the center of the hole and the center of the hole is an imaginary point. You cannot grab the center of a hole with your calipers. You can only grab the edge of a hole. So, the machinist has to measure from the edge of the part to the edge of the hole. Then he has to add half the hole's diameter to this number. That calculation will give the distance from the edge to the part to the center of the hole. The drawing below shows a plate dimensioned in millimeters:



At the top left there is a 36.57 dimension. This is what the distance from the left edge of the part to the center of the first hole should be. To verify this dimension, you have to measure the hole diameter with calipers, then measure from the left

edge of the part to the left edge of the hole. Then you have to add to the edge-to-edge distance, half of the hole diameter that you just measured.

Example:

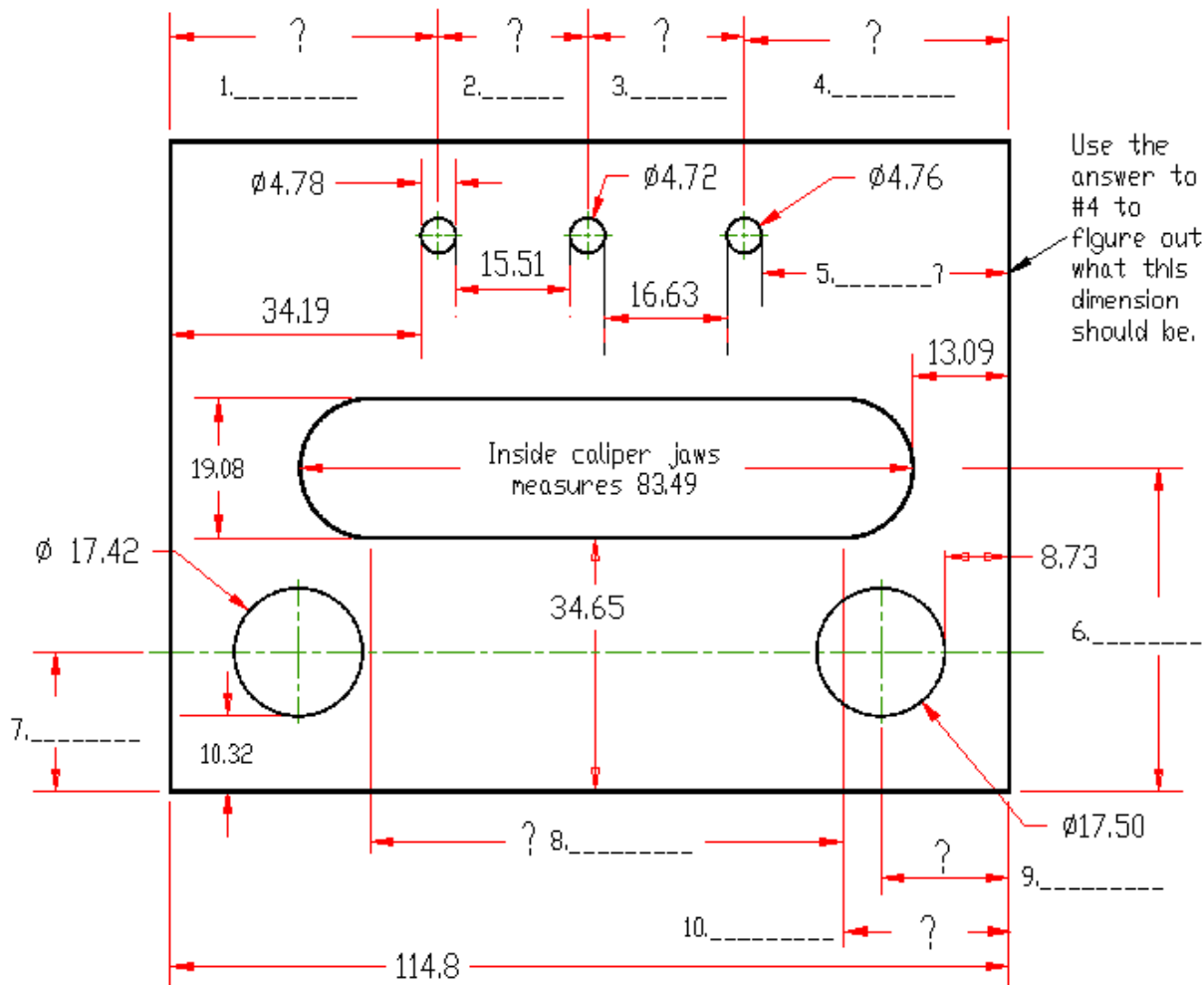
Suppose the hole measures 4.77 mm across and that the distance from the left side of the part to the edge of the hole measures 34.18. We then add half the hole diameter to 34.18,

$$34.18 + \frac{4.77}{2} = 34.18 + 2.385 = 36.56$$

which rounds off to 36.57. This is the distance from the left side of the part to the center of the hole.

The worksheet below is practice for figuring out hole locations as you would on the job. It gives dimensions for hole sizes and edge-to-edge distances. Like a real part, these dimensions are not exactly the same on the worksheet as on the print.

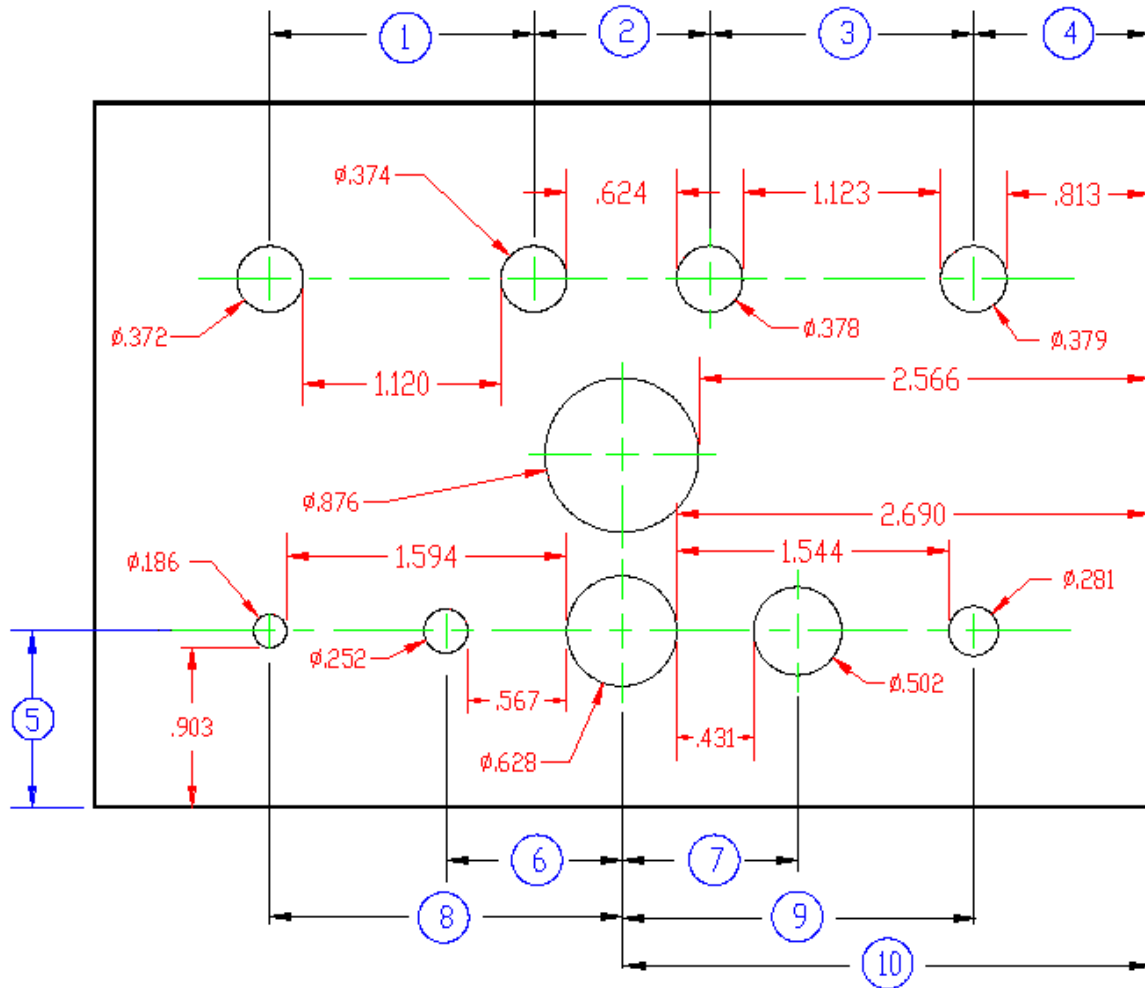
Use the hole diameters and the edge-to-edge distances to calculate the actual locations for dimensions 1 through 10.





### Worksheet 1-3: Caliper Math with 3-Place Decimals

Fill in the missing dimensions 1 through 10. Round off to 3 places.



(1) \_\_\_\_\_

(6) \_\_\_\_\_

(2) \_\_\_\_\_

(7) \_\_\_\_\_

(3) \_\_\_\_\_

(8) \_\_\_\_\_

(4) \_\_\_\_\_

(9) \_\_\_\_\_

(5) \_\_\_\_\_

(10) \_\_\_\_\_

## Worksheet 1-4: Four-Place Decimals

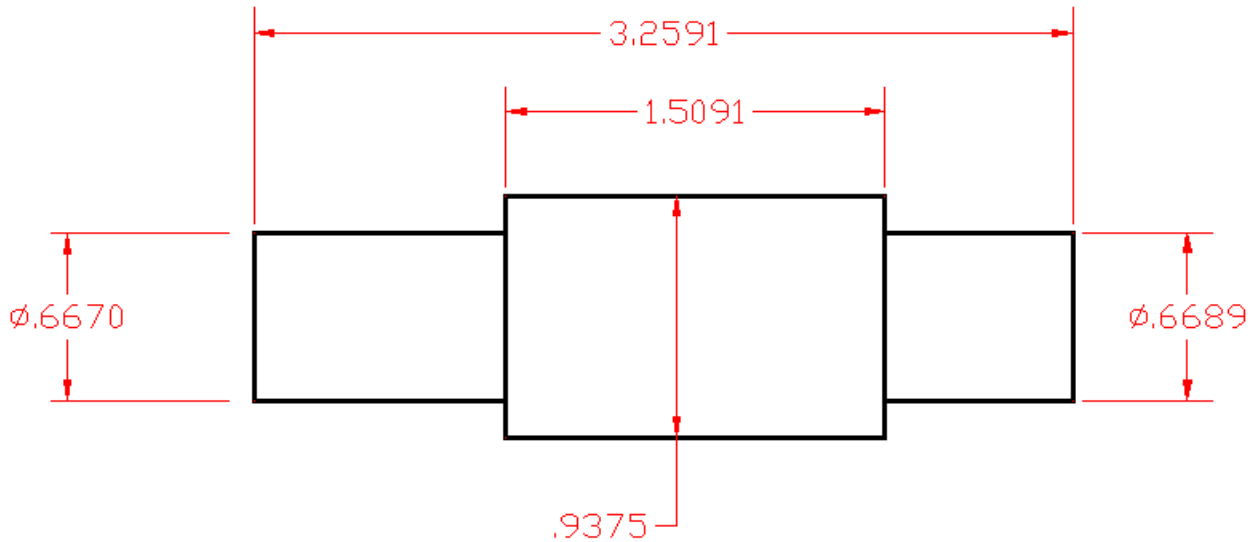
Parts that spin like rotors and crankshafts often require very close tolerances. These parts have dimensions with 4 and even 5 place decimals.

Sometimes a shaft is turned on a lathe and then ground to its final size on a cylindrical grinder. The lathe operator must not turn the part to the size given on the print. Instead, the lathe operator has to leave some stock for grinding.

How much stock should be left? That varies somewhat depending on the job but we will use a general guide of .005 to .010 stock left on the part for grinding. Note that on cylindrical parts, leaving .005" stock for the grinder means leaving a diameter that is .010" larger than what the print calls for.

Look at the drawings below. Recalculate each dimension so that an extra .0075 of material will be left for the grinding operation.

Note: If the dimension is marked with the diameter symbol ( $\varnothing$ ) you must add .015 to the dimension. For length dimensions, you just add .0075 to the print dimension. Give your answer as a number with four decimal places, even if the last decimal place is a zero.



1)  $3.2591 + .0075 =$

3.2666

2)

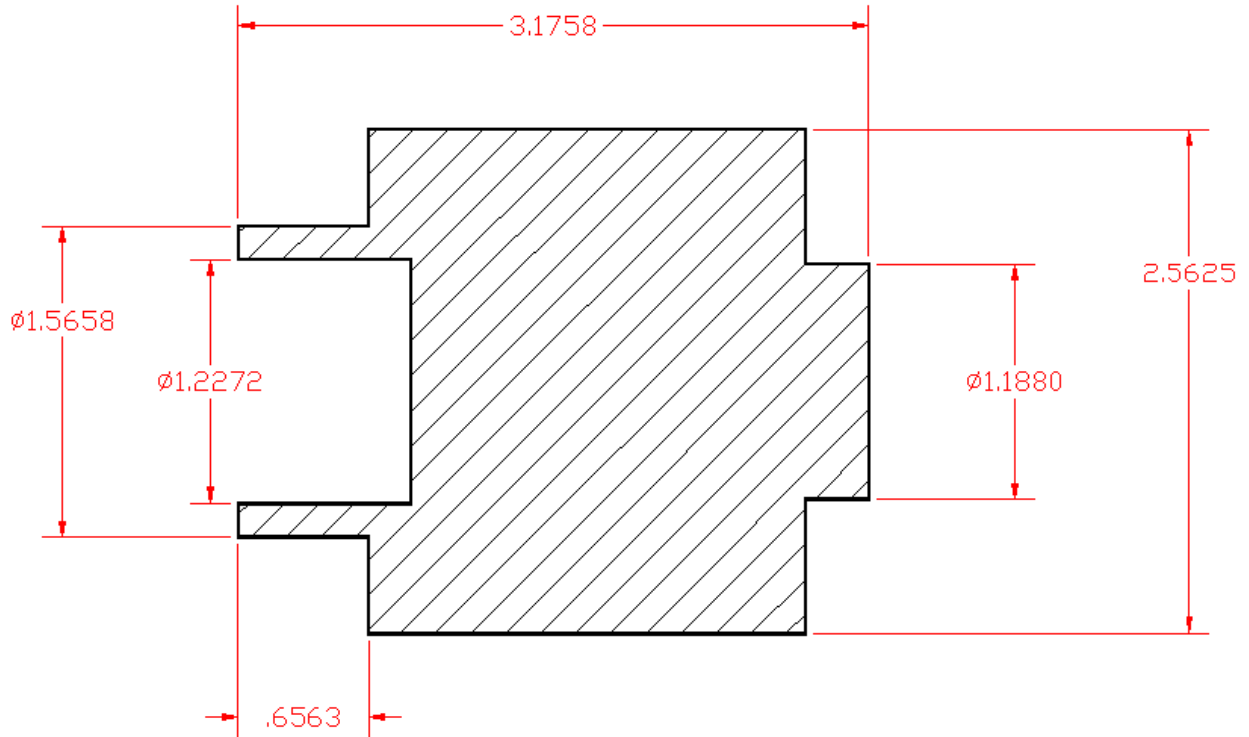
\_\_\_\_\_

4)

\_\_\_\_\_

5)

\_\_\_\_\_



1)  $\varnothing 1.5658 + .015 = 1.5808$

2) \_\_\_\_\_

3) \_\_\_\_\_

4) \_\_\_\_\_

5) \_\_\_\_\_

6) \_\_\_\_\_

7) \_\_\_\_\_

# Unit 2: Fractions

## MEET IT

A fraction is a way of expressing a value that is different from a decimal. All fractions are expressed in terms of one whole number (or integer) divided by another whole number. When you encounter a quantity expressed with a fraction, you will need to translate it in order to work in the decimal language.



When you translate a word from another language into English, you find the English word whose meaning most closely equals the meaning of the word you are translating. Sometimes the words mean exactly the same thing, and other times the meanings are not exactly the same, but as close as you can get. Similarly, to translate fractions to decimal language, you need to find the decimal number that most closely equals the fraction. Sometimes the two quantities will be exactly equal, and other times you have to determine the closest decimal number.

Safar is a machinist at a company that produces custom made capacitors for industrial applications. Safar is cutting brass on a vertical milling machine. “We often measure to within thousandths of an inch with micrometers and calipers, but when we set up a job, we have to locate the vise on the table with a scale (or ruler). So you have to know fractions as well as decimals.”

Sometimes the drawings or specifications give numbers in fractions, but Safar’s measurement tools, calculator, and computer all use a different language: decimals. You can see that to do his job well, Safar not only needs to have good knowledge of machine operations, procedures, and machining techniques, but he also must be able to accurately convert fractions to decimals. His quick ability to correctly convert fractions to decimals using a scientific calculator is of the utmost importance for his success on the

## LEARN IT

In this unit you will learn to:

- Use the rules for fraction addition, subtraction, multiplication and division
- Read fractional numbers from a ruler
- Convert fractions from prints to decimals using decimal equivalents charts
- Perform multi-step caliper math using fractions

## USE IT

Rulers and Scales

- 8ths and 16ths
- 32nds and 64ths (4R)
- 50ths and 100ths (16R)

Fraction arithmetic

- caliper math with fractions
- converting fractions to decimals
- sawing bar stock

## LEARN IT: READING A 6" SCALE

In machine shops, rulers are called scales. They come graduated in different fractions of an inch, or different "gradations." So, reading a scale is reading fractions. Note that the fractions on the scales we use in metalworking are only a small set of all the fractions. We don't use  $1/3$ 's or  $1/5$ 's. We only use  $1/2$ 's,  $1/4$ 's,  $1/8$ 's,  $1/16$ 's,  $1/32$ 's and  $1/64$ 's.

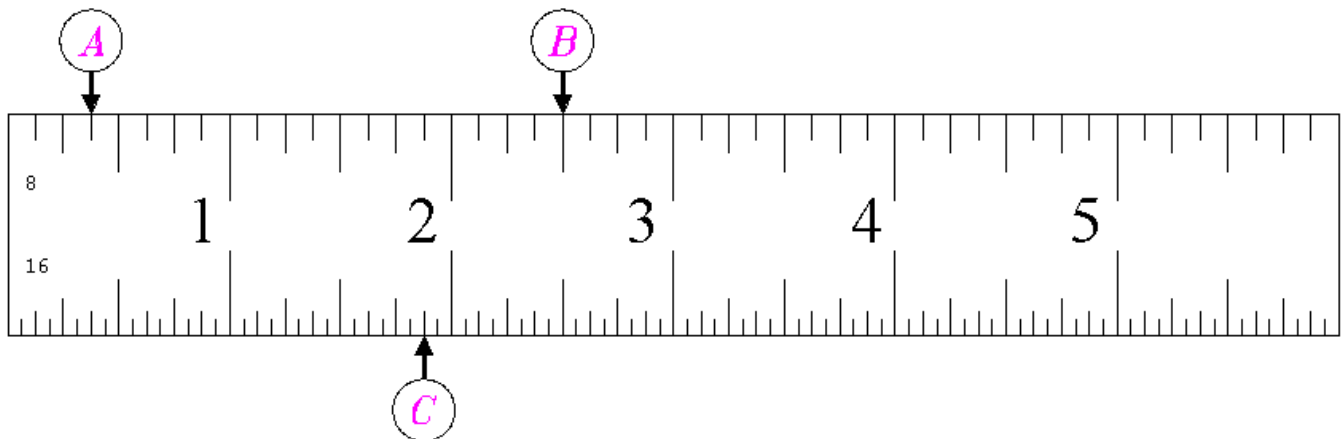
A ruler (or scale) that has 8ths and 16ths on one side, and 32nds and 64ths of an inch on the other side is called a 4R scale.

There are also scales graduated in  $1/50$ 's and  $1/100$ 's of an inch. A 16R scale has 32nds and 64ths of an inch on one side, and 50ths and 100ths on the other.

Look at the 6 inch scale below. It is graduated in  $1/8$ ths of an inch on the top side and  $1/16$ ths of an inch on the bottom.

What is the scale reading for the marker labeled "A"? You count the lines from the end. The "A" is at the 3rd line so it is reading  $3/8$  of an inch.

Remember that every ruler fraction has to have an odd number on top or it is not in lowest terms. The "B" marker is at the 4th line so it is  $4/8$ , which is the same as  $1/2$ . The reading for "B" is 2 and  $1/2$  inches. Remember to include the whole number and not just the fraction in your answers!

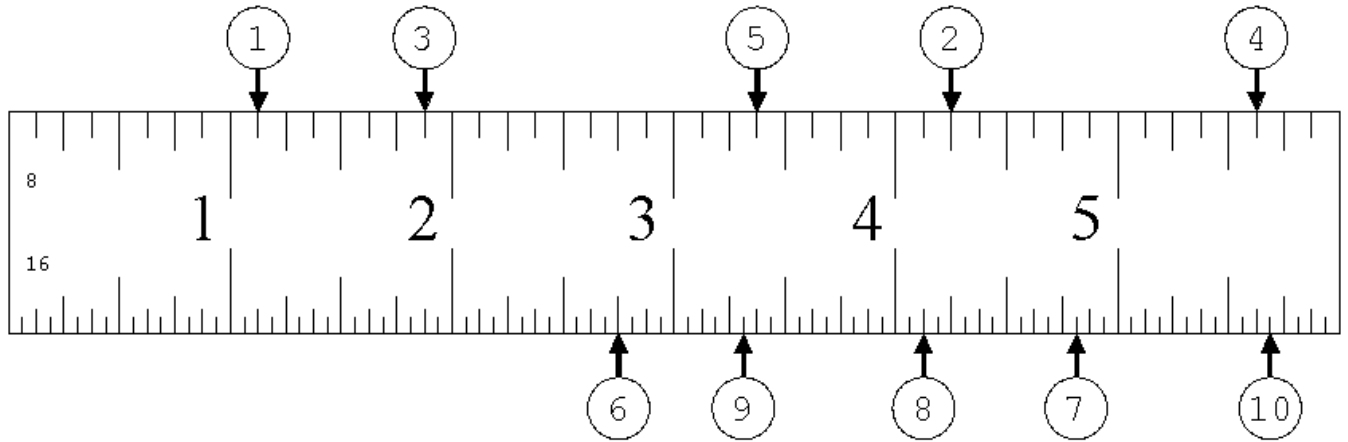


The bottom of the ruler is graduated in  $1/16$ ths of an inch. Count the lines to take a reading but "count the lines odd"!

Marker C is 14 lines past 1, so that is 1 and  $14/16$  or 1 and  $7/8$  inches. Can you think of a way to read the ruler without having to count every little line?

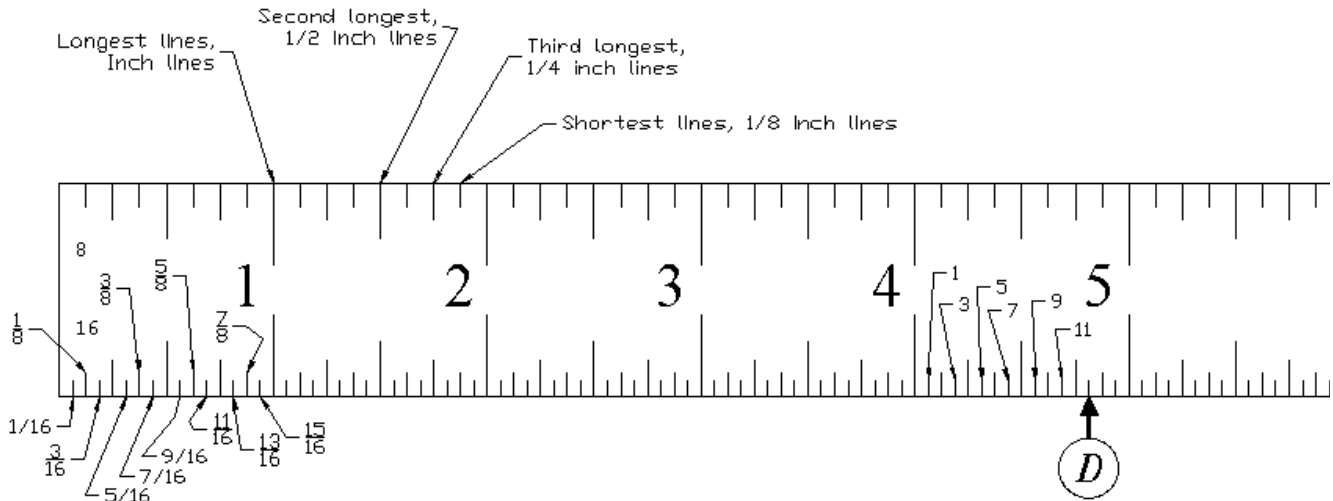
## Worksheet 2-1: Reading 8ths and 16ths of an Inch

Write down the readings for markers 1 through 10 on the answer spaces below. Give your answer as fractions reduced to their lowest terms.



- 1) \_\_\_\_\_
- 2) \_\_\_\_\_
- 3) \_\_\_\_\_
- 4) \_\_\_\_\_
- 5) \_\_\_\_\_
- 6) \_\_\_\_\_
- 7) \_\_\_\_\_
- 8) \_\_\_\_\_
- 9) \_\_\_\_\_
- 10) \_\_\_\_\_

## LEARN IT: How to Count the lines odd.



Look at the line at marker "D". Decide which type of line it is touching, is it a 16th or 8th or what.?

This is the smallest type of line so it is a 16th line. Now count from the nearest inch line but count only these 16th lines and count by odd numbers: 1, 3, 5, etc.

What did you get? 13, so marker "D" reads 4 and 13/16 inches.

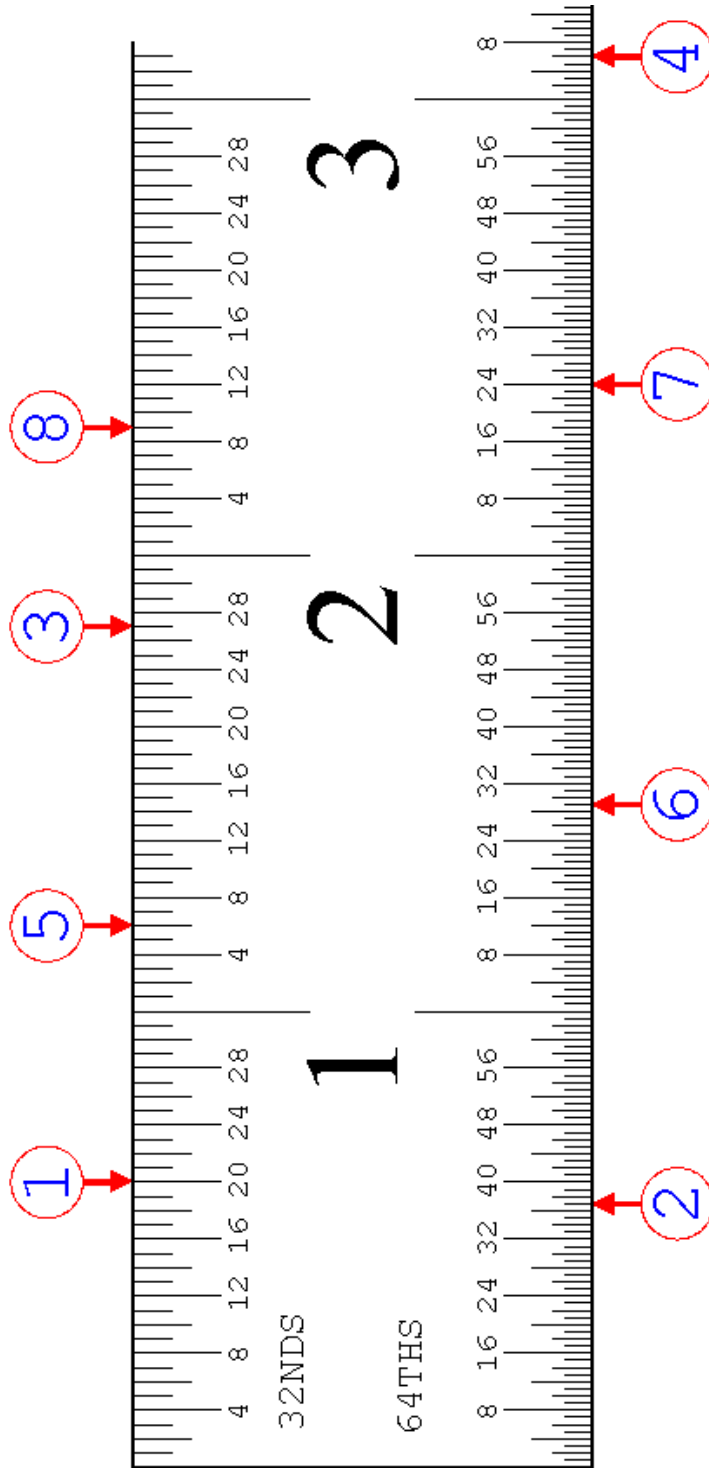
When you have trouble taking readings for scales graduated in 16ths, 32nds or 64ths, try counting the lines odd:

- 1) Decide which type of line you have to read.
- 2) Count ONLY those types of line and COUNT them ODD.



## Worksheet 2-2: Reading a Scale with 32nds and 64ths

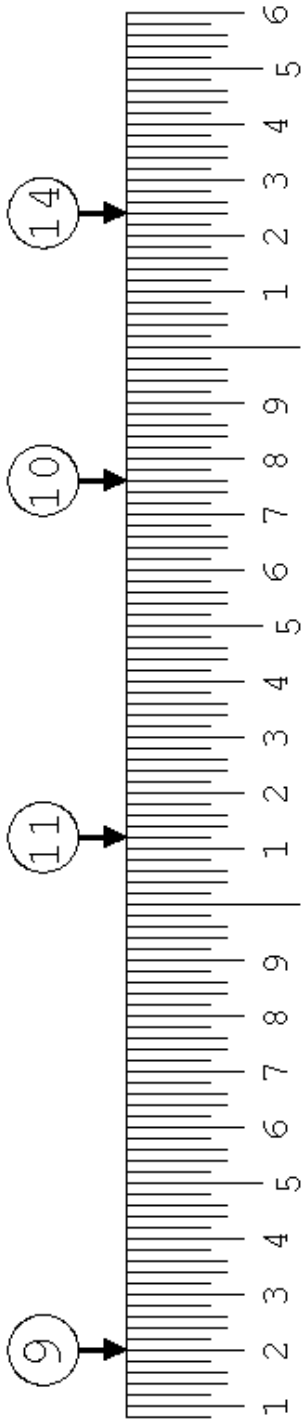
**WORKSHEET 2-2: READING A SCALE WITH 32NDS AND 64THS**



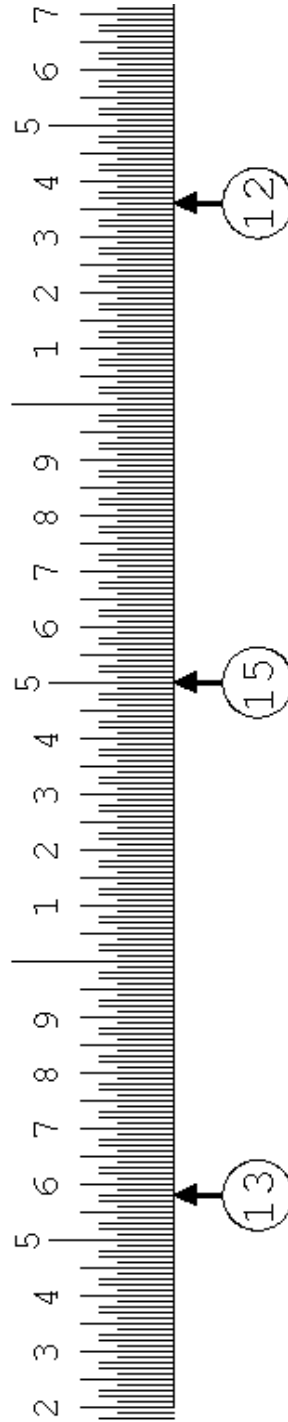
- 1) \_\_\_\_\_
- 2) \_\_\_\_\_ 3) \_\_\_\_\_ 4) \_\_\_\_\_
- 5) \_\_\_\_\_ 6) \_\_\_\_\_ 7) \_\_\_\_\_ 8) \_\_\_\_\_

### Worksheet 2-3: 16R Scale, 50ths and 100ths

#### WORKSHEET 2-3: 16R SCALE: 50THS AND 100THS



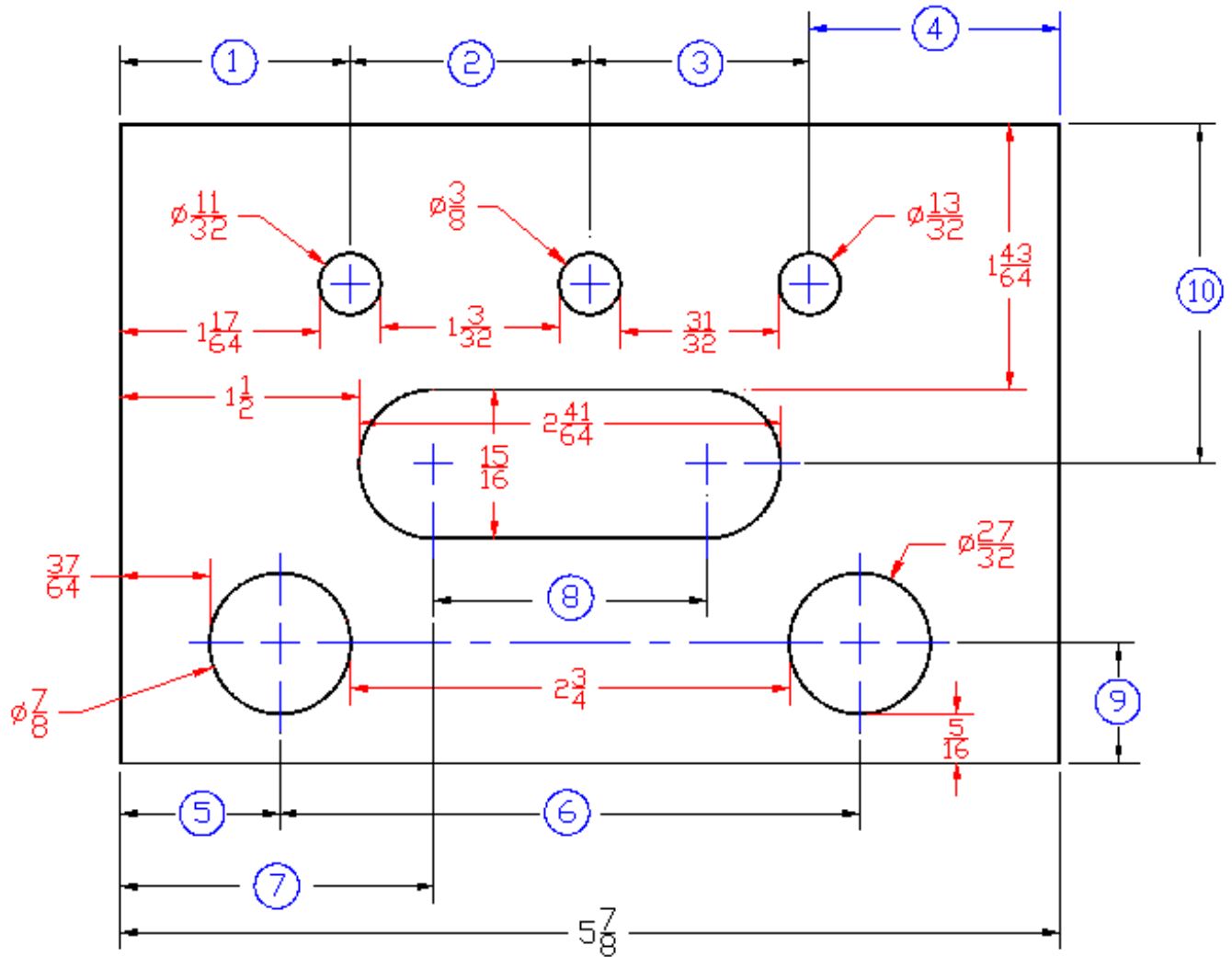
2 3



9) \_\_\_\_\_ 10) \_\_\_\_\_ 11) \_\_\_\_\_

12) \_\_\_\_\_ 13) \_\_\_\_\_ 14) \_\_\_\_\_ 15) \_\_\_\_\_

### Worksheet 2-4: Hole Locations with Fractions



(1) \_\_\_\_\_

(6) \_\_\_\_\_

(2) \_\_\_\_\_

(7) \_\_\_\_\_

(3) \_\_\_\_\_

(8) \_\_\_\_\_

(4) \_\_\_\_\_

(9) \_\_\_\_\_

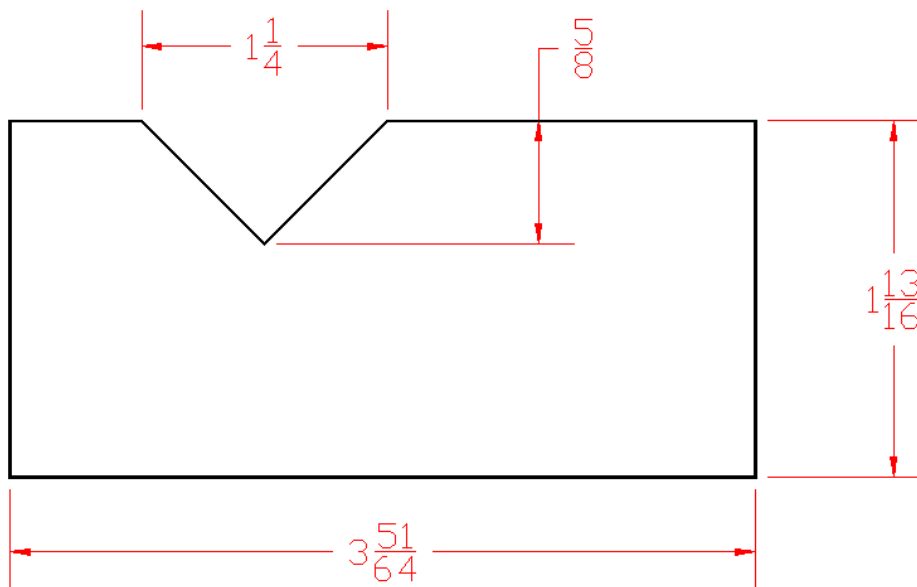
(5) \_\_\_\_\_

(10) \_\_\_\_\_

## LEARN IT: USING DECIMAL EQUIVALENTS CHARTS

Fractions are used only occasionally on prints today. Many companies do not use fractions at all on prints. Even where fractions are used on the print, it is often a good practice to convert all the fractions shown on the print to decimals. You can convert the fractions manually with paper and pencil, or with a calculator, or by using a decimal equivalents chart. To use the chart, look up the fraction you need to convert and the decimal number next to it is the equivalent. The chart below shows these decimal equivalents to 3 places. This is usually enough decimal places for fractional dimensions.

Example: Look at the drawing below and use the chart on the next page to finish the examples.



1)  $1\frac{1}{4} = 1.250$  (look up  $\frac{1}{4}$  which is  $.250$ , don't forget the whole numbers in your answer!)

2)  $\frac{5}{8} =$  \_\_\_\_\_

3)  $1\frac{13}{16} =$  \_\_\_\_\_

4)  $3\frac{51}{64} =$  \_\_\_\_\_

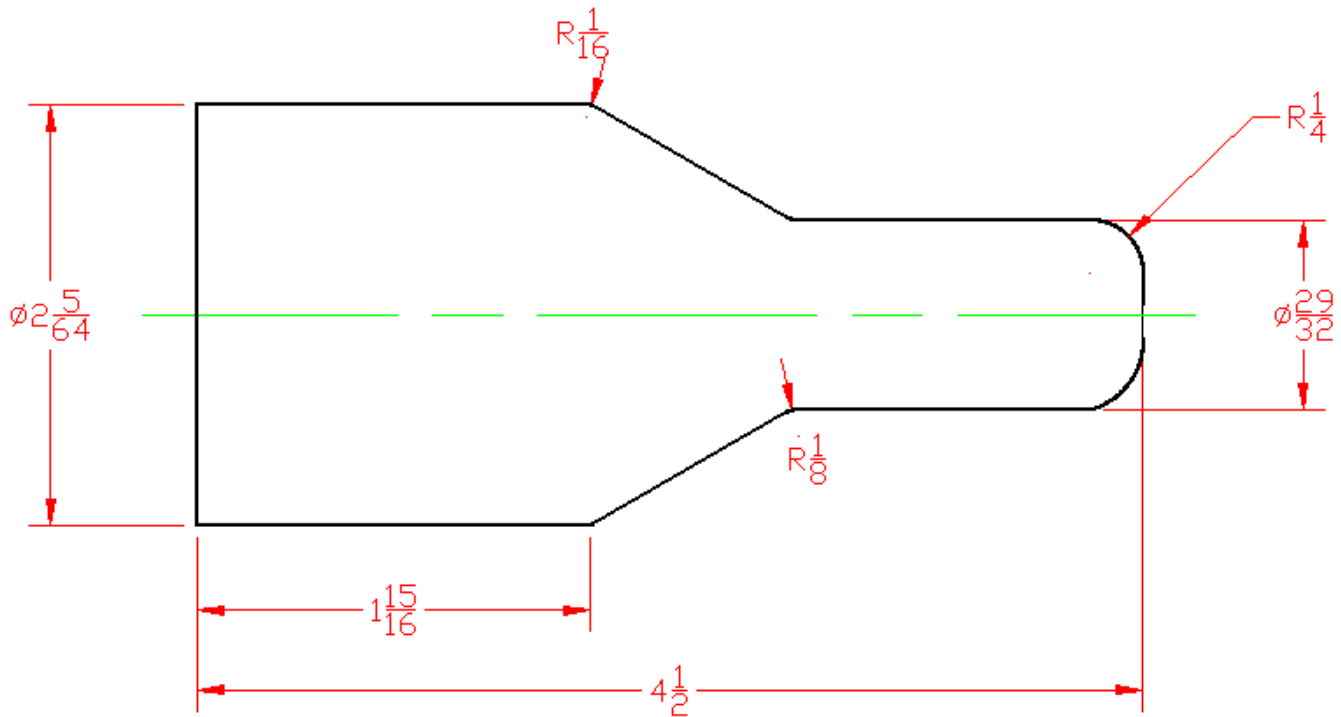
**DECIMAL EQUIVALENTS**

(Rounded to 3 Decimal Places)

1/64	.016	33/64	.516
1/32	.031	17/32	.531
3/64	.047	35/64	.547
1/16	.063	9/16	.563
5/64	.078	37/64	.578
3/32	.094	19/32	.594
7/64	.109	39/64	.609
1/8	.125	5/8	.625
9/64	.141	41/64	.641
5/32	.156	21/32	.656
11/64	.172	43/64	.672
3/16	.188	11/16	.688
13/64	.203	45/64	.703
7/32	.219	23/32	.719
15/64	.234	47/64	.734
1/4	.250	3/4	.750
17/64	.266	49/64	.766
9/32	.281	25/32	.781
19/64	.297	51/64	.797
5/16	.313	13/16	.813
21/64	.328	53/64	.828
11/32	.344	27/32	.844
23/64	.359	55/64	.859
3/8	.375	7/8	.875
25/64	.391	57/64	.891
13/32	.406	29/32	.906
27/64	.422	59/64	.922
7/16	.438	15/16	.938
29/64	.453	61/41	.953
15/32	.469	31/32	.969
31/64	.484	63/64	.984
1/2	.500	1	1.000

## Worksheet 2-5: Converting Fractions to Decimals on a Print

Now convert all the fractions on the print below to their decimal equivalents. Use the chart, not a calculator.



(1)  $1) R \frac{1}{16} = .063$

(2) \_\_\_\_\_

(3) \_\_\_\_\_

(4) \_\_\_\_\_

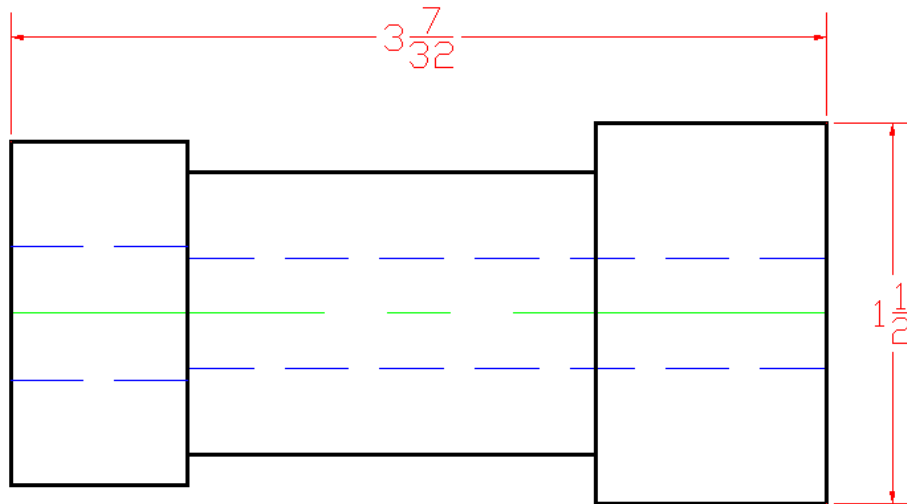
(5) \_\_\_\_\_

(6) \_\_\_\_\_

(7) \_\_\_\_\_

## Worksheet 2-6: Sawing Bar Stock

A job order calls for slicing bars of metal into “blanks” that can be machined later on mills and lathes. The job order doesn’t say how long you have to make the “blanks” but you have a print that gives the final dimensions for the part.



The overall length of the part is  $3 \frac{7}{32}$ . To this you must add the width of the saw blade (typically  $\frac{1}{16}$ ”), and you must also leave some material for “clean up.” In this case, leave an extra  $\frac{1}{8}$ ” to remove the saw cuts on both ends of the part. For this part, you would set the stop on your saw to cut pieces of length:

$$3 \frac{7}{32} + \frac{1}{16} + \frac{1}{8} = 3 \frac{13}{32}$$

With the same allowances for the saw width and clean up, what length would you cut blanks for jobs where the overall part length is:

$$2 \frac{1}{2}$$

$$9 \frac{13}{16}$$

What length would you cut if the saw was  $\frac{1}{8}$ ” wide and you had to leave  $\frac{3}{16}$  for clean up and the print gave an overall length of:

$$6 \frac{3}{16}$$

$$5 \frac{11}{32}$$

# Unit 3: Tolerances

## MEET IT

We plan many things in life, but actually very few of them turn out absolutely perfectly or exactly as we plan them. They usually turn out good enough, and we accept the result. Another way to say this is that we tolerate it when things are close enough to our plan, even if they're not perfect.

Precision metalworking involves making metal parts according to somebody's plan. Although we try to make every part perfectly, we know that every aspect of every part does not turn out exactly as the plan calls for. That's why the people who make the plans tell us the maximum deviation they can tolerate, so we know how close to perfect we need to get. This is called tolerance.

Karen is a milling machine operator. She uses drawings, layouts, and set-up instructions to set up and operate horizontal and vertical milling machines. She knows each of her machines and what it can do. The old vertical mill sometimes has trouble holding a setting, and may take off too much material. Karen has learned to compensate for this by setting the machine "half a line high" and frequently measuring the work. She has to pay special attention when a counterbore tolerance is very small or "tight," because it is very difficult to keep the old machine on track. "It's like driving my old Mustang," she says. "The steering is kind of loose and the car wanders back and forth in the lane. When I'm on the open road it's OK, because I have a lot of tolerance on where I can drive without hitting anyone. But when there's a lot of close traffic, the tolerance is a lot less, and I get a little nervous."

Araceli, on the other hand, operates a CNC cylindrical grinder. This machine can grind to size within millionths of an inch. However, even with this type of precision, there is still a tolerance applied to the dimensions. The main difference is that Araceli is working with tolerances of 4 or 5 decimal places while Karen works with 3-place decimals.





## LEARN IT

In this unit you will learn to:

- Understand dimensions and tolerances
- Understand special and standard tolerances
- Distinguish between bilateral, unilateral, and limit tolerances
- Calculate the limits of decimal and fractional tolerances

## USE IT

- Calculate limits for dimensions with fractions and 2- and 3- place decimals
- Calculate limits for dimensions with special tolerances
- Calculate limits for 3- and 4- place decimals from print dimensions

## LEARN IT: Dimensions vs. Tolerances

### A. Definitions

Dimension refers to the specified nominal size of a feature on the part. This is the number written on the print. Dimensions may be expressed as fractions or as decimals. Some dimensions are more critical than others.

Tolerance refers to how far you can stray from the nominal size of a feature. Once you know the tolerance, you can calculate your lower and upper limits. Anything that measures outside of these limits would be undersize or oversize (U/S or O/S).

### B. Dimension vs. Tolerance

In a perfect world, every single feature on every single part would be right on the nose. In the real world, we experience some variation from part to part. For example, if we inspected a .312 dimension on five random parts, we might get the following readings: .306, .310, .311, .312, .307. Only one of the five parts is right at nominal (.312). Are the other four parts bad?

To answer this question, we have to know how much tolerance is allowed for this dimension. If we are allowed plus or minus five thousandths of an inch (written on the print as “+/- .005” or as “±.005”), then four parts are good and one is bad. However, if we are allowed ten thousandths of an inch (+/- .010 or ±.010), then all five parts are good.

Again, some dimensions are more critical than others. The amount of tolerance allowed for a given dimension is determined by the functional importance of that feature of the part. Imagine that you are producing shelves for prefabricated bookcases. The length and width of the shelf are more critical than the thickness of the shelf. (The length and width of the shelves could affect the correct assembly of the bookcase. Nobody will notice that one shelf is slightly thicker than another.)

Every dimension has a specified tolerance. Based on the tolerance, we can infer the lower and upper control limits for that feature. We can understand the relative functional importance of that feature! Let's look at how at how this information is presented in blueprints.

## C. Fraction and Decimal Dimensions

As you know, dimensions may be presented as fractions or as decimals on a blueprint. To make things even more confusing, decimal dimensions are sometimes written with two decimal places, sometimes three. So, a “half inch” dimension could be written a number of different ways:

$$\frac{1}{2} \qquad .50 \qquad .500$$

The same dimension can be written three different ways. Why? The answer is that how a dimension is written communicates information about the tolerance. Look at the sample tolerance section of a blueprint on the next page. Look at the sample tolerance block below. Note that on a blueprint, this tolerance block generally appears next to or inside the title block

---

Unless otherwise specified, tolerances are as follows.

3-Place Decimals  $\pm .005$   
 2-Place Decimals  $\pm .010$   
 Fractional  $\pm 1/64$

---

This explains the mystery. To an experienced machinist, “ $\pm .005$ ” reads as “plus or minus .005 of an inch.” Thus:

**.500** means the part must measure between five hundred thousandths of an inch plus five more thousandths and five hundred thousandths of an inch minus five thousandths:

$$\begin{array}{r} .500 \\ - .005 \\ \hline .495 \text{ is the lower limit} \end{array} \qquad \begin{array}{r} .500 \\ + .005 \\ \hline .505 \text{ is the upper limit.} \end{array}$$

**.50** means five hundred thousandths of an inch plus or minus ten thousandths.

$$\begin{array}{r} .500 \\ - .010 \\ \hline .490 \text{ is the lower limit} \end{array} \qquad \begin{array}{r} .500 \\ + .010 \\ \hline .510 \text{ is the upper limit.} \end{array}$$

$1/2$  means  $1/2$  plus or minus  $1/64$  of an inch.

$$\frac{1}{2} - \frac{1}{64} = \frac{31}{64} \text{ is the lower limit.} \quad \frac{1}{2} + \frac{1}{64} = \frac{33}{64} \text{ is the upper limit.}$$

In every case, the dimension was five hundred thousandths of an inch (.500). How the dimension is presented (.500, .50,  $1/2$ ) tells us how much tolerance is permissible. This tells us which features of the part are most critical, which features demand the most precision.

It tells us something else. It allows us to plan our workday. Some parts have dozens, even hundreds of dimensions. It is humanly impossible to check every dimension on every part 100% of the time and still meet production requirements in a manufacturing environment. So, an experienced machinist knows to pay more attention to dimensions, which have tighter tolerance.

At the start of your shift, always check your first part 100% and make any adjustments necessary to bring all dimensions closer to nominal. (Warning: Don't take my word that the job is running good. You are responsible! You have to check it yourself!) After you check the first part 100%, you will check some dimensions more frequently than others. A threeplace decimal (.500) requires constant attention. However, a two-place decimal (.50) may be checked once or twice an hour. Fractional dimensions might only be checked only on the first piece. The operations sheet or the in-process inspection sheet will tell you how often to check dimensions.

Fractional dimensions can turn into decimals.

Sometimes the tolerance for a fractional dimension is given as a decimal. Instead of  $\pm 1/64$  the tolerance might be  $\pm .015$  or  $\pm .030$ . In this case you have to convert the fraction dimension to a decimal and then calculate the upper and lower limits. For example:

$1/2$  means five hundred thousandths of an inch plus or minus fifteen thousandths.

$$\begin{array}{r} .500 \\ - .015 \\ \hline .485 \text{ is the lower limit} \end{array} \quad \begin{array}{r} .500 \\ + .015 \\ \hline .515 \text{ is the upper limit} \end{array}$$

Note that  $1/64$  converted to a decimal is .015625 which when rounded off to 3 decimal places would be .016". Since this is a tolerance, we have to be cautious so machine shops round down to .015. Generally, machinists think of  $\pm 1/64$  as  $\pm .015$ .

Note that  $1/64$  converted to a decimal is  $.015625$  which when rounded off to 3 decimal places would be  $.016$ ". Since this is a tolerance, we have to be cautious so machine shops round down to  $.015$ . Generally, machinists think of  $\pm 1/64$  as  $\pm .015$ .

Why would the tolerance on a fraction be given as a decimal number? One reason is to make everyone think in terms of decimals and not fractions. Keep in mind that micrometers and calipers give readings as 3-place decimals. Since we measure in thousandths of an inch, we have to think of dimensions as thousandths of an inch even when they are given as fractions.

Then why use fractions at all? Because a fraction is another form to write a number and it is more precise than a 1-place decimal. We can get four different standard tolerances if we use fractions along with 2, 3, and 4-place decimals.

One more thing, the tolerance blocks look different on different prints. Some will look like this:

---

Unless otherwise specified, tolerances are as follows.

Fractions	$\pm .015$
.XX	$\pm .01$
.XXX	$\pm .005$
.XXXX	$\pm .0005$

---

Here,  $.XX$  means a two-place decimal,  $.XXX$  means a three-place decimal, and  $.XXXX$  means, you guessed it, a four-place decimal.

On a metric print, the tolerance block is likely to look like this:

---

Dimensions are in millimeters.  
Unless otherwise specified, tolerances are as follows.

X.X	$\pm 0.1$
X.XX	$\pm 0.05$

---

With inch dimensions we always drop the leading zero, for example:  $.125$

However, with metric dimensions we always include the leading zero:  $0.79$

## Worksheet 3-1: 2-Place, 3-Place and Fractional Tolerances

Instructions. Calculate the upper and lower limits for these dimensions. Use your calculator.

3 Place Decimals Dimensions ± .005	2 Place Decimals ± .01	Fractional ± .015
Example 1 .500 .505/.495	.50 .510/.490	1/2 .515/.485
Example 2 .312 .317/.307	.31 .320/.300	5/16 .328/.298

3 Place Decimals ± .005	2 Place Decimals ± .010	Fractional Dimensions ± .015
(1) .250	.25	1/4
(2) .750	.75	3/4
(3) .125	.13	1/8
(4) .625	.63	5/8
(5) .375	.38	3/8
(6) .563	.56	9/16
(7) .438	.44	7/16
(8) .938	.94	15/16
(9) .813	.81	13/16
(10) .343	.34	11/32

**HOT TIPS:**

(1) The most common mistake occurs in the middle column. Keep in mind that plus or minus ten thousandths of an inch is .01 not .10. This is a common error, which occurs on the floor when operators make offsets in CNC programs. Even when the print says: “.XX ±.01”, you should think “.XX ±.010”.

(2) On fractional dimensions, your answers may vary by a thousandth (.001) depending on whether you round up or down. That’s OK.

## Worksheet 3-2: Special Tolerances

The titleblock lists the standard tolerances. However, there are many special tolerances to contend with. Note that the tolerance section of the blueprint says, “unless otherwise specified.” That means, “unless we tell you differently, use the standard tolerances listed in the titleblock.”

Frequently, the standard tolerances cover most of the dimensions, but critical dimensions often have tighter tolerances (for example,  $.500 \pm .002$ ) and will likely use special tolerances.

As well, some dimensions are less fussy than standard so it makes sense to give you more than the standard tolerance. For example, the dimensions you use to locate a name plate on a fixture might have a special tolerance of  $\pm 1/16$ .

Using special tolerances can save money as many tolerances can be increased over the standard while only a few need to be tighter. It is not surprising that some companies have more special than standard tolerances on their prints.

Calculate the lower and upper limits for these special dimensions. Use your calculator.

Example 1                       $.500 \pm .002$                        $.502/.498$

Example 2                       $.312 \begin{smallmatrix} +.002 \\ -.001 \end{smallmatrix}$                        $.314/.311$

Problems:

$.250 \pm .003$                        $.406 \pm .003$

$.625 \begin{smallmatrix} +.001 \\ -.005 \end{smallmatrix}$                        $.100 \begin{smallmatrix} +.010 \\ -.005 \end{smallmatrix}$

$.8761 \pm .0003$                        $.438 \pm .0015$

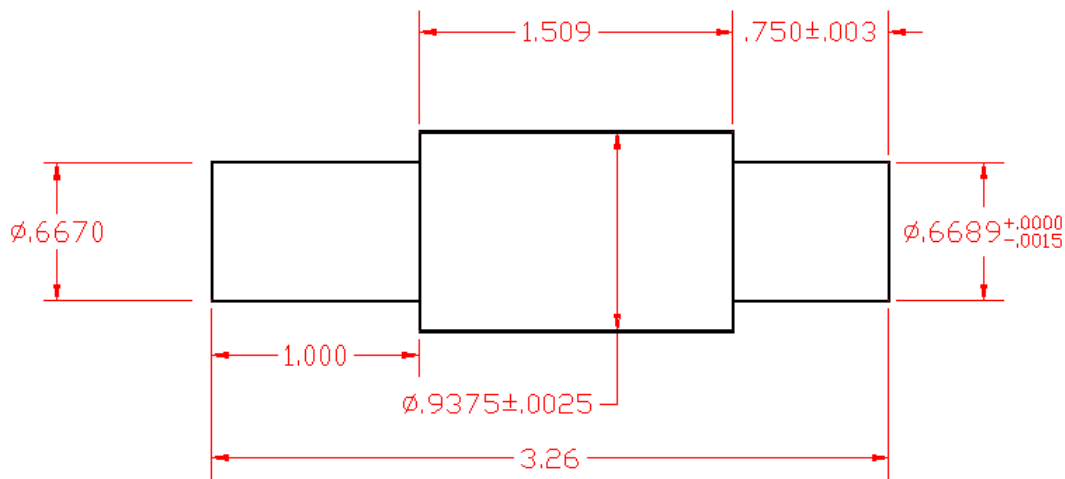
$3.2 \pm 0.12$                        $\frac{15}{32} \pm \frac{3}{62}$

### Worksheet 3-3: Print Tolerances of 2, 3, and 4 Place Decimals

Calculate the upper and lower limit for each dimension on the drawings below.

Dimensions are in inches. Unless otherwise specified, tolerances are:

.XX	±.01
.XXX	±.005
.XXXX	±.0005
Fractions	±1/32



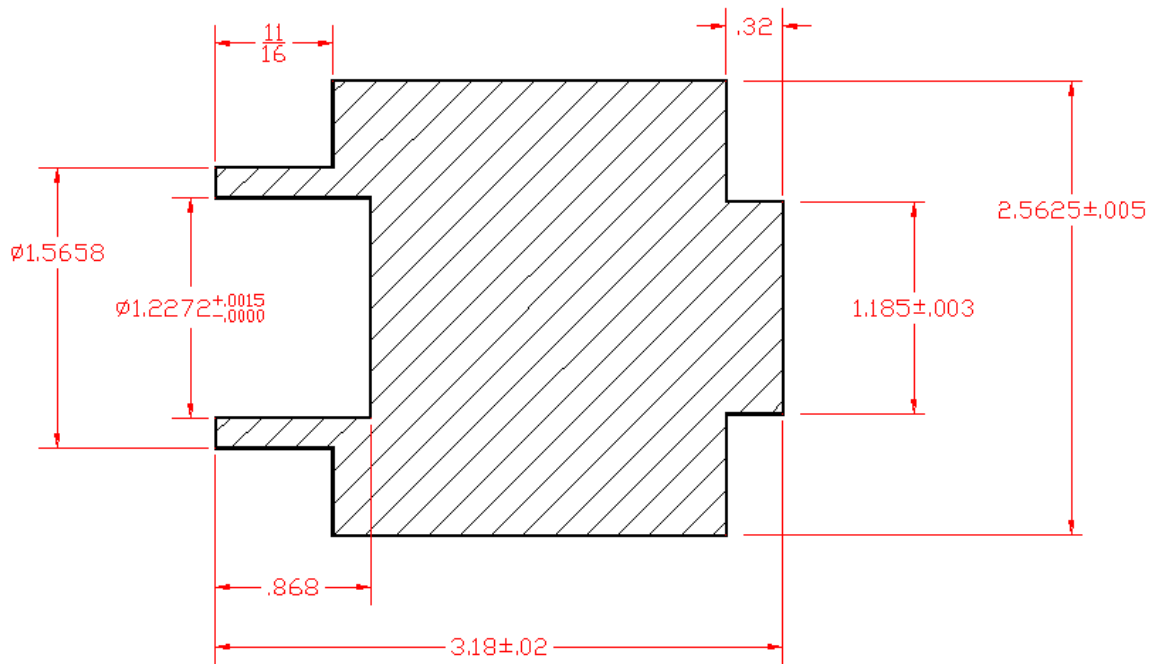
- |                        |                   |                        |                   |
|------------------------|-------------------|------------------------|-------------------|
| 1) 1.509               | Upper Limit _____ | 5) 1.000               | Upper Limit _____ |
|                        | Lower Limit _____ |                        | Lower Limit _____ |
| 2) .0750               | Upper Limit _____ | 6) $\varnothing$ .9375 | Upper Limit _____ |
|                        | Lower Limit _____ |                        | Lower Limit _____ |
| 3) $\varnothing$ .6670 | Upper Limit _____ | 7) 3.26                | Upper Limit _____ |
|                        | Lower Limit _____ |                        | Lower Limit _____ |
| 4) $\varnothing$ .6689 | Upper Limit _____ |                        |                   |
|                        | Lower Limit _____ |                        |                   |



Calculate the upper and lower limit for each dimension on the drawings below.

Dimensions are in inches. Unless otherwise specified, tolerances are:

.XX	±.01
.XXX	±.005
.XXXX	±.0005
Fractions	±1/32



- |                    |                   |           |                   |
|--------------------|-------------------|-----------|-------------------|
| 1) $\frac{11}{16}$ | Upper Limit _____ | 5) 1.185  | Upper Limit _____ |
|                    | Lower Limit _____ |           | Lower Limit _____ |
| 2) .32             | Upper Limit _____ | 6) 2.5625 | Upper Limit _____ |
|                    | Lower Limit _____ |           | Lower Limit _____ |
| 3) $\phi 1.5658$   | Upper Limit _____ | 7) .868   | Upper Limit _____ |
|                    | Lower Limit _____ |           | Lower Limit _____ |
| 4) $\phi 1.2272$   | Upper Limit _____ | 8) 3.18   | Upper Limit _____ |
|                    | Lower Limit _____ |           | Lower Limit _____ |

# Unit 4: Percents

## MEET IT

Percents are often used to indicate a fraction, portion, or “share” of something. For example, another way of saying one-half is “50 percent.” The sales tax on a new shirt may be 8 percent (the government’s “share”). Your share of a winning lottery ticket you bought with three friends would be 25 percent.



Percents also give us a convenient way to compare. The sale price on a CD might be a 20 percent “off” the regular price. A new pain capsule may work ten percent faster than aspirin. A new design for a part maybe 40 percent lighter than the old design.

Terry is a gage maker at a company that makes electrical components. “We use ‘fixed gages’ to verify hole sizes and locations in our parts, so we can make sure the parts will fit together. For example, I make GO and NO-GO pins to check hole sizes (see photo). A machinist making a part can get the tolerances right off the print. Well, I can’t do that because the print doesn’t say anything about making gages. So I have to calculate a ‘gage maker’s tolerance’ that’s a percentage of the print tolerance. I take 10% of the total tolerance that’s shown on the print for that particular dimension, and sometimes add another 5% of that for wear. I do these percent calculations all the time, so now I can do most of them in my head.”

## LEARN IT

In this unit you will learn to:

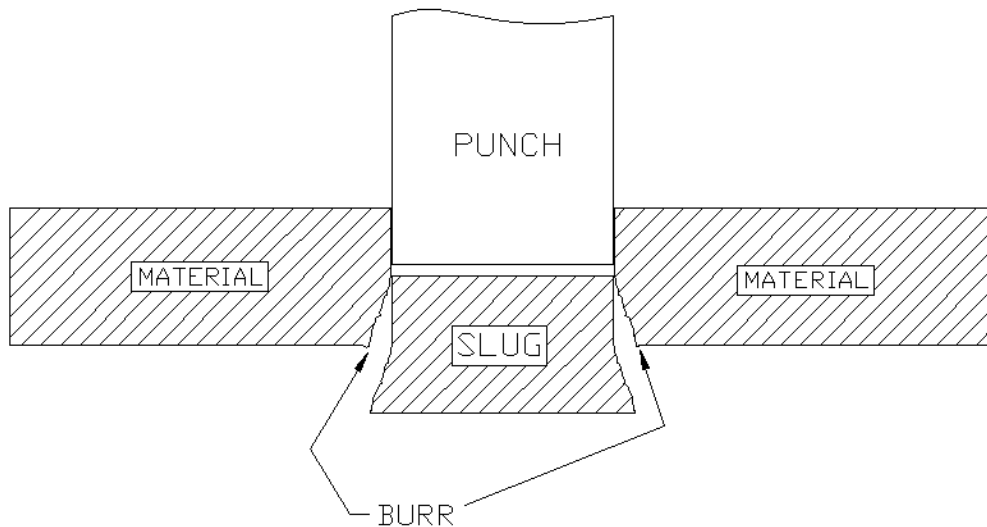
- Calculate tolerances as a portion of a dimension
- Add and subtract percentages to dimensions

## USE IT

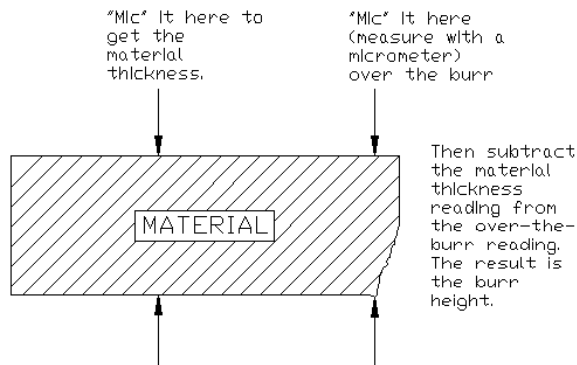
- Calculate allowable burr size on stampings (5% of material thickness)
- Calculate size tolerance for a “GO/NO-GO” gage (10% of tolerance on dimension being gaged, plus 5% more for wear)

# WORKSHEET 4-1: Allowable Burr Size on Stampings

Punched holes always have a small burr around them on the side where the slug breaks through.



If this burr is too large it can cause problems. Therefore, the press operator has to measure it with micrometers and decide if it is acceptable.



How much burr is allowed? A rule of thumb is that the height of the burr cannot be greater than 5% of the material thickness.

So, if you are punching holes in sheet metal that is .062" thick, the maximum allowable burr is:

$$.062" \times 5\% = .062" \times .05 = .0031"$$

Calculate the maximum allowable burr height for material that is:

.016" thick

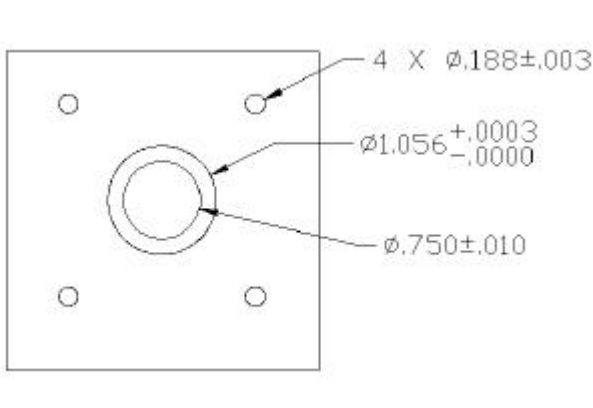
.036" thick

.042" thick

.010" thick

## Worksheet 4-2: Gagemaker's Tolerance for a GO/NO-GO Gage

To make gage pins for the part shown below, you have to calculate 10% of the total tolerance for each dimension.



For the Ø.188, the tolerance is ±.003 so the total tolerance is .006. We need to find 10% of .006.

$$.006 \times 10\% = .006" \times .1 = .0006$$

Now the gage maker will use .0006 as the total tolerance on the gage pins that he is making. The operator will use these gage pins to check that the hole size is within tolerance.

The lower limit is  $.188 - .003 = .185$ , this is the GO pin.

The gage maker will make a GO pin that falls between a minimum size of .185 and a maximum size of .1856"

The upper limit is  $.188 + .003 = .191$ , this is the NO-GO pin. The NO-GO pin will measure between a maximum of .191 and a minimum of  $.191 - .0006 = .1904$ . So, for the Ø.188 hole, the GO Pin limits are .185/.1856; the NO-GO limits are .1904/.191.

Calculate the maximum and minimum limits for GO/NOGO pins for the other two diameters on this part:

Ø.750 GO Pin: \_\_\_\_\_ / \_\_\_\_\_

NOGO pin: \_\_\_\_\_ / \_\_\_\_\_

Ø1.056 GO Pin: \_\_\_\_\_ / \_\_\_\_\_

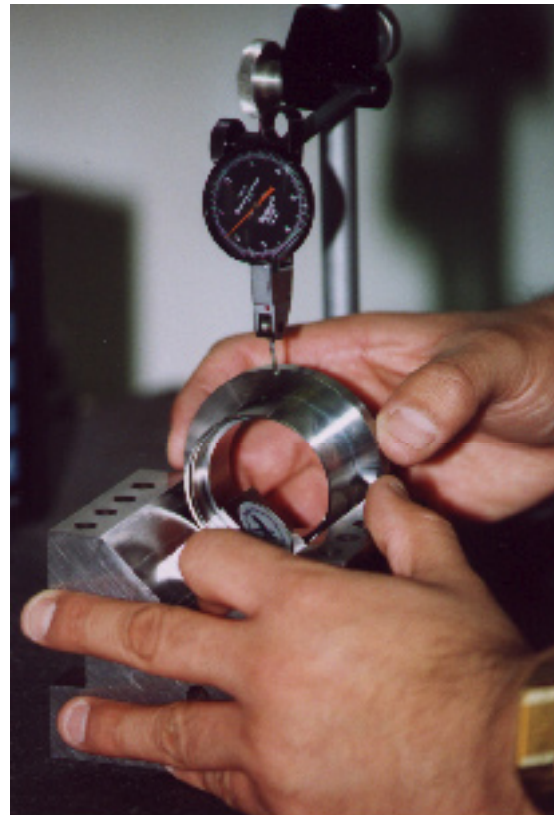
NOGO pin: \_\_\_\_\_ / \_\_\_\_\_

# Unit 5: Angles and Triangles

## MEET IT

Drafting is the skill of drawing an object with enough information, detail, and precision to allow someone to make the object. Prints and shop drawings are examples of drafting. Creating and interpreting these drawings requires an understanding of geometry. Simple geometry includes rules about the way straight lines come together to form various shapes, such as squares, rectangles, and triangles, and the measurements associated with those shapes.

Prints do not usually provide every dimension of the object they describe. Sometimes you need to figure out a dimension from the drawing by using the rules of geometry. Drawings are made with circles, straight lines, angles, and shapes such as rectangles and triangles. Understanding basic geometry will give you a clearer mental picture of the object as you review the print.



Checking the axial runout of the flange with a dial indicator

Hector is a floor inspector in a CNC machine shop. He reads complicated drawings and specifications for parts that are to be made, then determines if the “first article” produced by the operators are acceptable. Hector uses scales, squares, micrometers, protractors, verniers, plug gages, height gages, dial indicators and optical comparators to verify print that the part conforms to the print requirements. Hector has an excellent understanding of basic geometry, and he’s glad he does. “We have to make precise angular measurements,” says Hector. “We also use a lot of GD&T (geometric dimensioning and tolerancing). If I didn’t know basic geometry, I couldn’t do the more complicated pieces at all, and even the easy ones would take all day.”

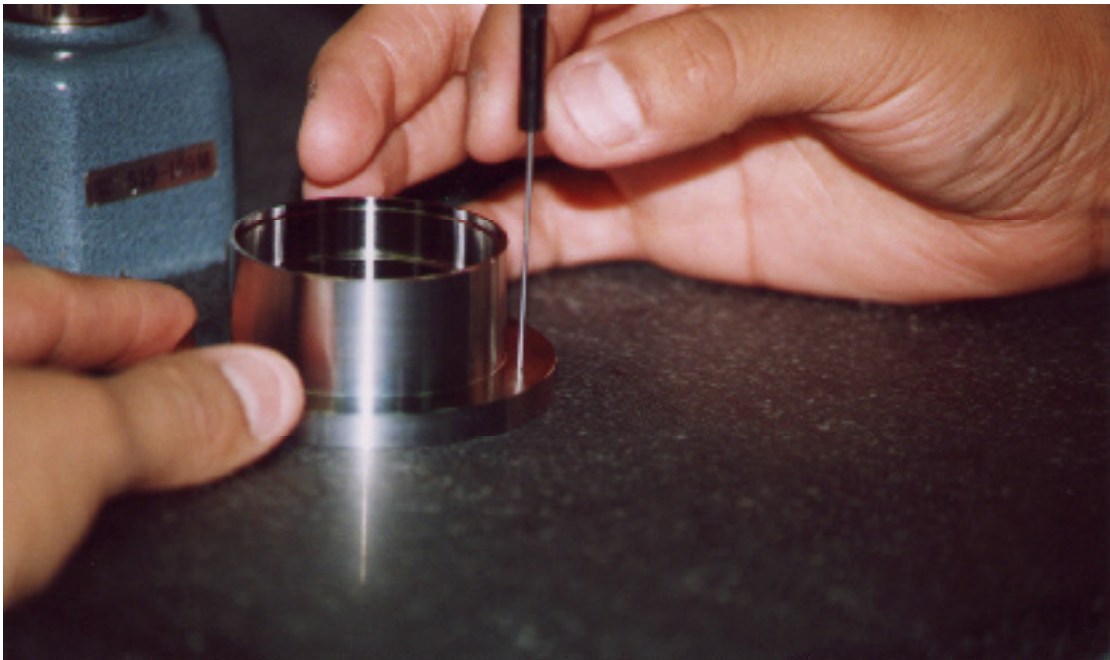
## LEARN IT

In this unit you will learn more about:

- Degrees and the circle
- Angles inside triangles
- Opposite angles and complimentary angles
- Some rules for relating angles on the print to part features
- Tolerances on angular dimensions

## USE IT

- Find values of complementary and supplementary angles
- Identifying angles on prints
- Angular tolerances

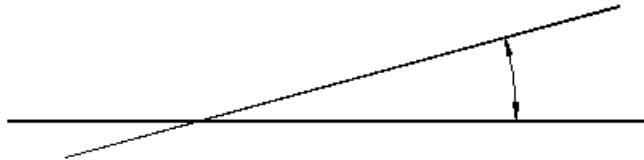


This inspector is using an electronic height gage to check parallelism of the top and bottom surfaces of the flange on this part.

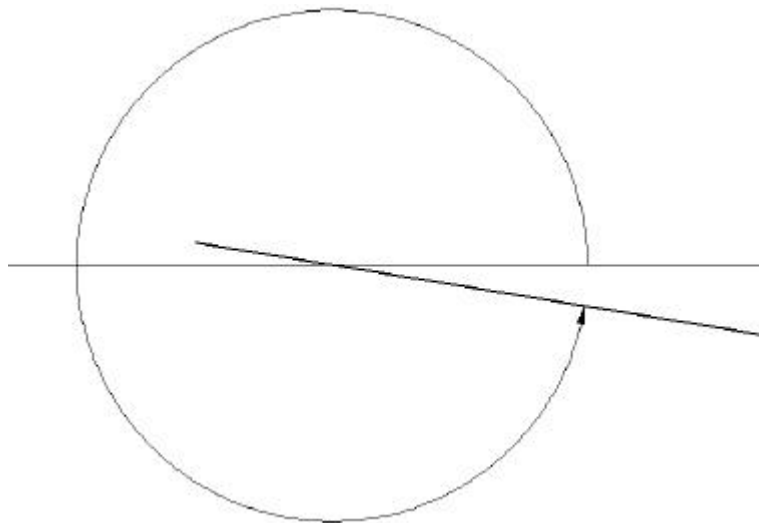


## LEARN IT:

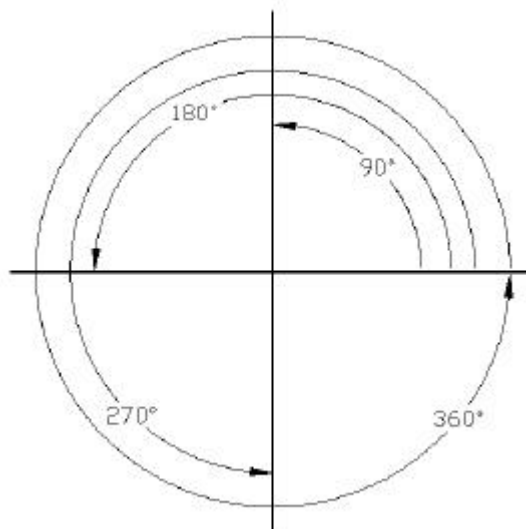
Think of angles as a measure of rotation. Nail two sticks together and swivel them apart:



The angle between them is a measure of how far one rotated compared to the other. If we spin one stick a full circle it is rotated 360 degrees.



If we rotate the stick  $\frac{1}{4}$  circle, then the angle of the two sticks is  $90^\circ$ , and so forth.



Please note that a half circle is  $180^\circ$ , which is also the angle of a straight line! A straight line describes an angle of  $180^\circ$ .

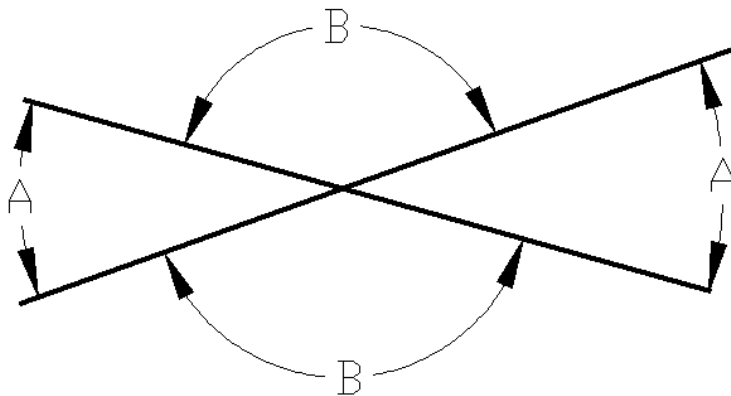
Of course, when the print calls for one surface to be at a 45-degree angle to another surface, we don't think of rotating the surface of the part to make the 45-degree angle.

Yet, when we measure this angle, we do swivel the straight-edge of our machinist protractor around the part to find the angle. So, it is helpful to think of angles as measurements of rotation.

## SOME FACTS ABOUT ANGLES:

Fact #1: Opposite angles are equal.

Imagine two sticks nailed together and swiveling or rotating around the nail. We can draw two lines to represent them:



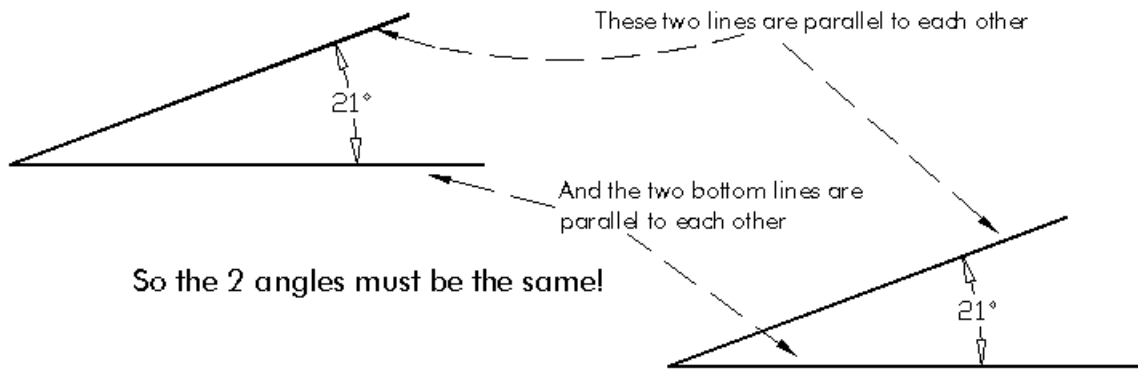
These lines depict four angles. However, the angles that are opposite to each other are equal. Think of a pair scissors. The more you open the handles, the more the scissors blades open up.

Fact #2: Supplementary angles equal  $180^\circ$ .

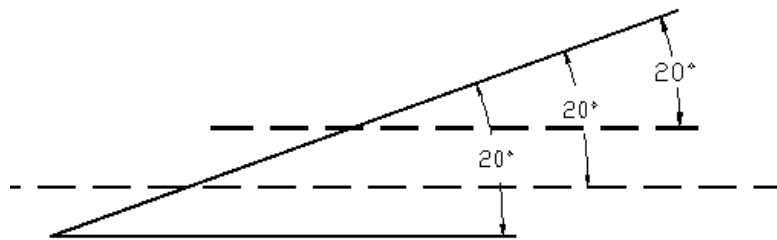
In the drawing above angles A and B are supplementary. Together they form a straight line and which is always  $180^\circ$ . If we know one of these two angles, we can always find the value of the other one. The math formula for this is:

$$A + B = 180^\circ \quad \text{and from this formula we can also get,}$$
$$A = 180^\circ - B$$

Fact #3: If the sides of two angles are parallel to the corresponding sides on another angle, then the angles must be the same. These are called “Corresponding Angles.”



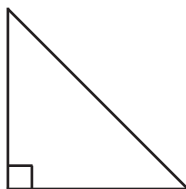
One way to show this is to draw an angle with one side horizontal and then draw more horizontal lines through it. All the horizontal lines are parallel to each other and the diagonal line is parallel to itself, so all the angles identified are equal.



Right Angles are Special.

An angle of  $90^\circ$  is called a right angle. When a line or a surface is at a  $90^\circ$  angle to another line or surface, then they are perpendicular to each other.

Right angles can be indicated by a little box in the corner:

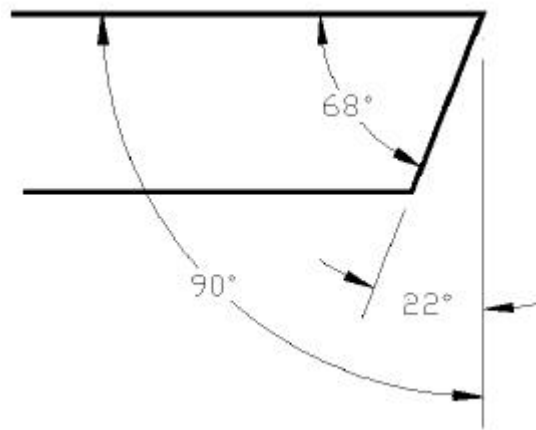


This marks the “square corner.” The tool used to check for the “squareness” of a corner, to make sure it is  $90^\circ$ , is called a “square.”

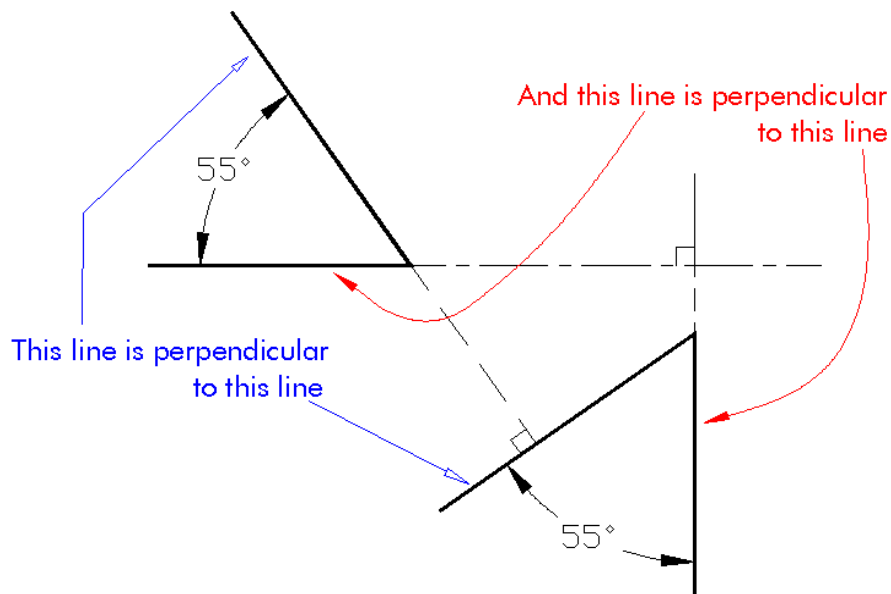
Any time you see a square corner of a blueprint, it is called an “implied” right angle. Even if though there is no angle dimension on the square corner, it must be  $90^\circ \pm$  the standard angular tolerance.

Fact #4: Complementary angles combine to form a right angle.

When two angles add up to  $90^\circ$ , they are said to be complementary angles. The drawing below shows two angles that add up to  $90^\circ$ .



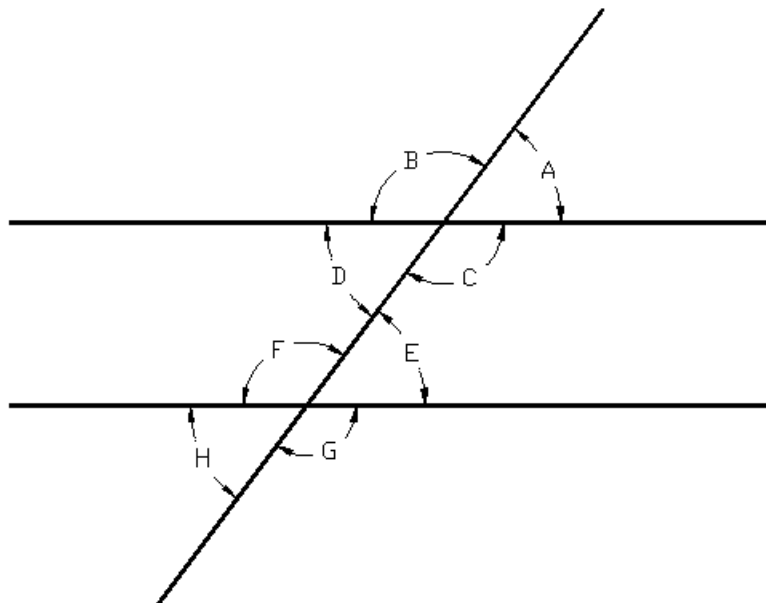
Fact #5: If each side of an angle is perpendicular to the lines defining another angle, then these two angles must also be equal.



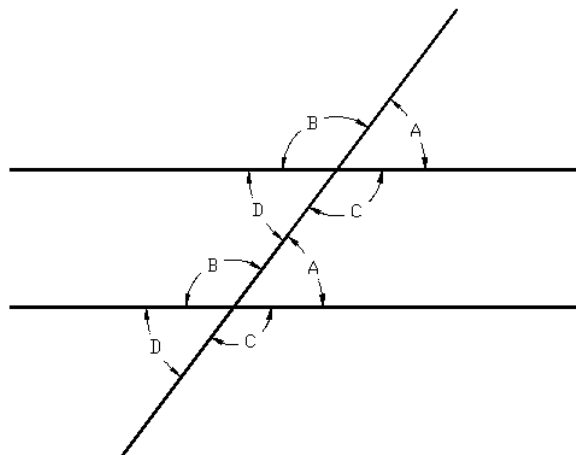
Fact #6: When two parallel lines have a third line drawn across them, the corresponding angles are equal.

Fact #7: The alternate exterior and alternate interior angles of these same lines are equal.

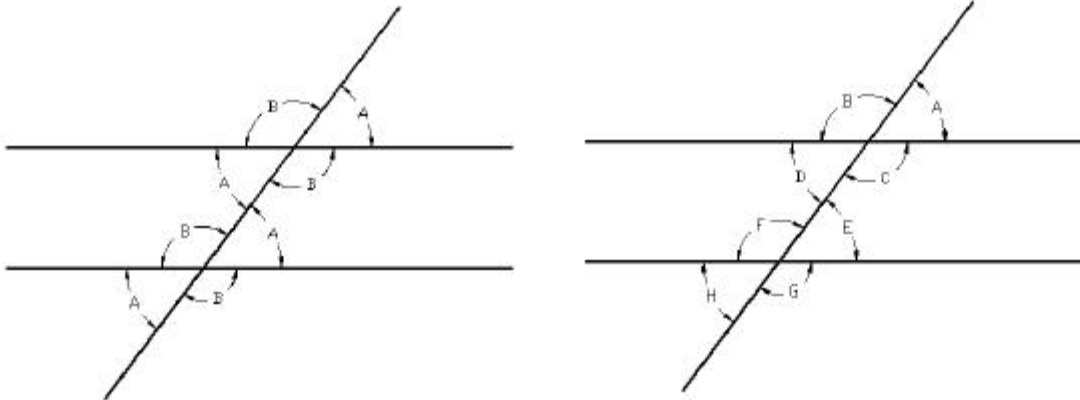
The drawing below shows two parallel lines with another straight line cutting across them. These 3 lines describe 8 angles, A through H. In this example, angles A and H are alternate exterior angles, as are B and G. D and E are alternate interior angles along with C and F.



Note that A corresponds to E, and B corresponds to F, and C to G and D to H (Fact #3). So now we have:



From Fact #1, we also know that opposite angles are equal so we have:



Fact #7 said that the alternate interior angles are equal and the alternate exterior angles are equal. Look at the two drawings above.

Angles D and E are alternate interior angles and they are both equal to angle A.

Angles C and F are alternate interior angles and they both equal angle B.

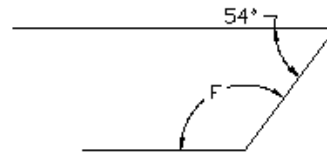
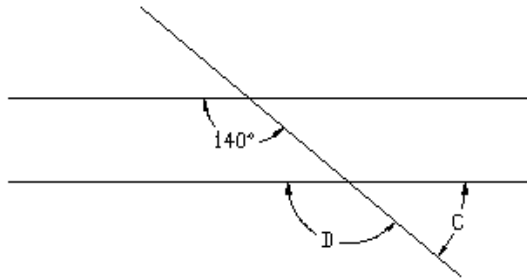
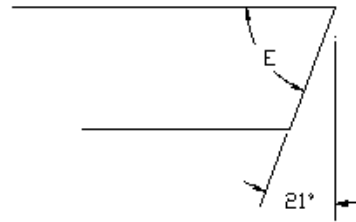
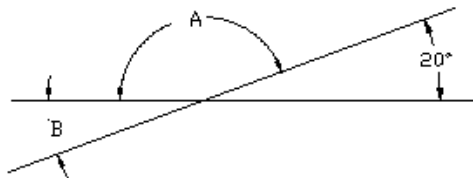
There are two pairs of alternate exterior angles, B and G; and A and H.

Does  $B=G$  and  $A=H$ ?

We started with 3 lines, 2 of which are parallel, describing 8 angles but there are only 2 different angles, A and B. Further, if we know the value of one of these, we can calculate the other because  $A+B=180^\circ$ . So, if we just know one of these angles, we can figure out all the others.

## Worksheet 5-1: Complementary and Supplementary Angles

In the drawing below, figure out the value of angles A through F.



A \_\_\_\_\_ B \_\_\_\_\_ C \_\_\_\_\_ D \_\_\_\_\_ E \_\_\_\_\_ F \_\_\_\_\_

Hints for E and F:

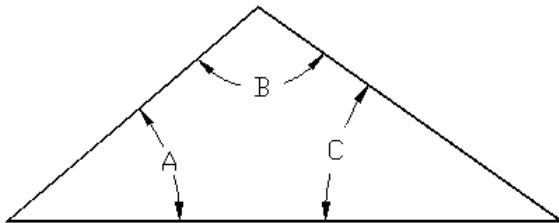
If you look closely, you can see that angle E and  $21^\circ$  totals up to  $90^\circ$ . Also, you can get angle F if you extend the horizontal lines and figure it as two parallel lines cut by

# LEARN IT

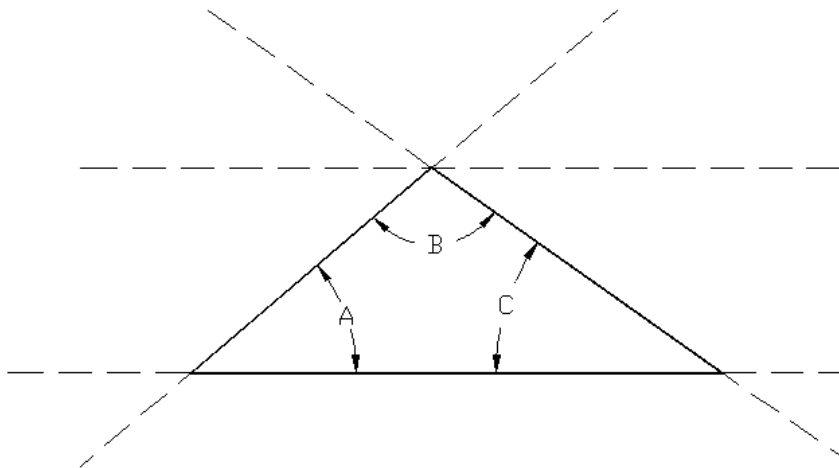
## Angles in Triangles

Triangle fact #1:

If you add up all the angles inside a triangle, how many degrees do you get?  $180^\circ$ ! The proof is below.



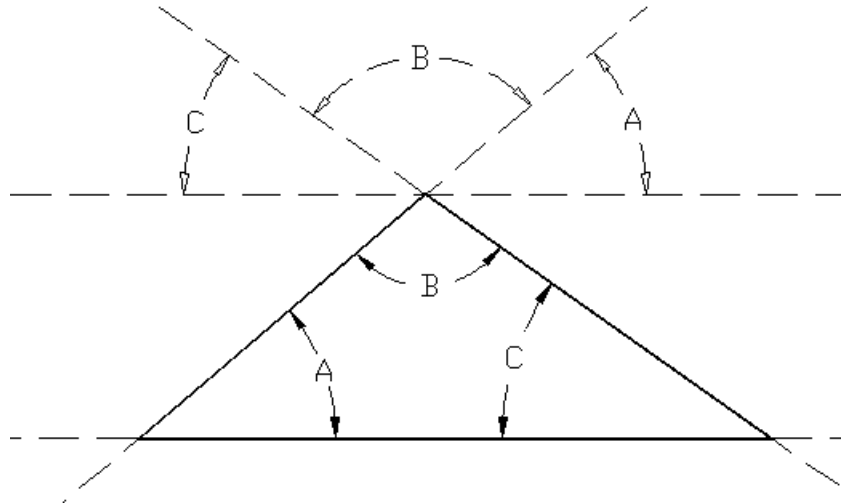
First we have a triangle. It has three internal angles. We don't know what they are and we don't have to. All we have to show is that  $A+B+C=180^\circ$ . To do this we will draw a horizontal line through the top of the triangle. This line is parallel to the base of the triangle. We will also extend the sides of the triangle above and below these lines. What we get is:



Do you see how to use angle facts #1 and #5 to prove that  $A+B+C= 180^\circ$ ?

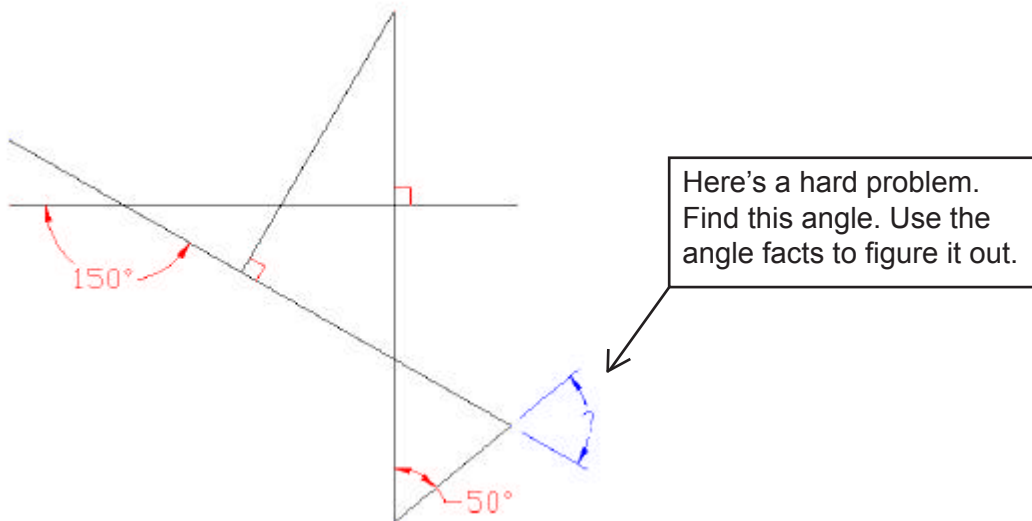


Because opposite angles are equal, and because corresponding angles are equal, we can show angles A, B, and C at the top.



This also shows that angles  $C+B+A$  define a straight line. A straight line =  $180^\circ$

Therefore, we have proved that the interior angles of a triangle add up to  $180^\circ$ !

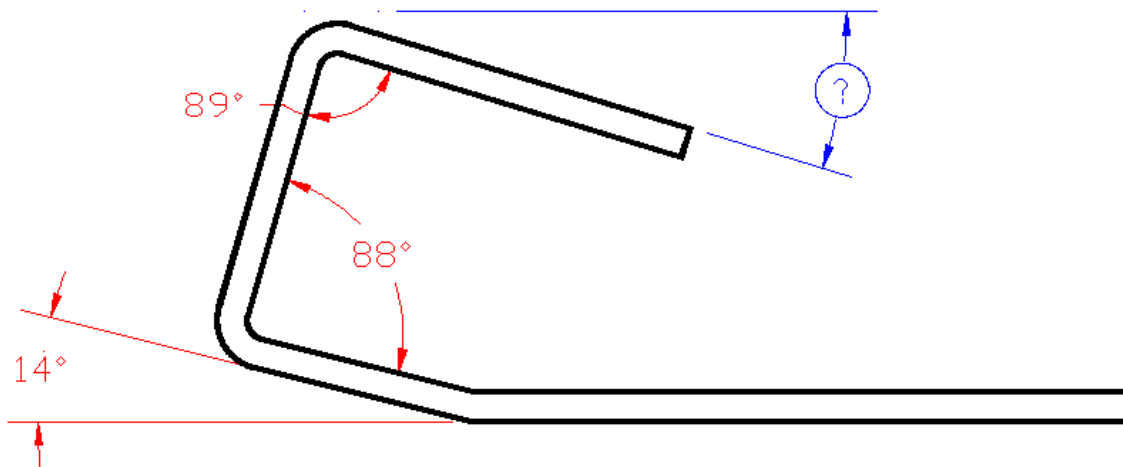
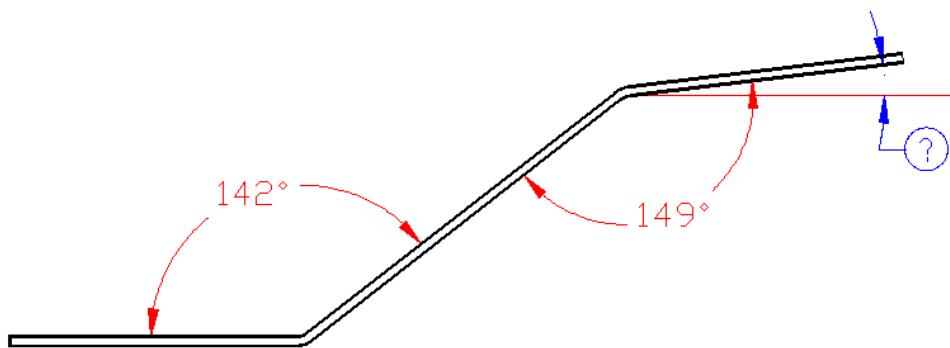


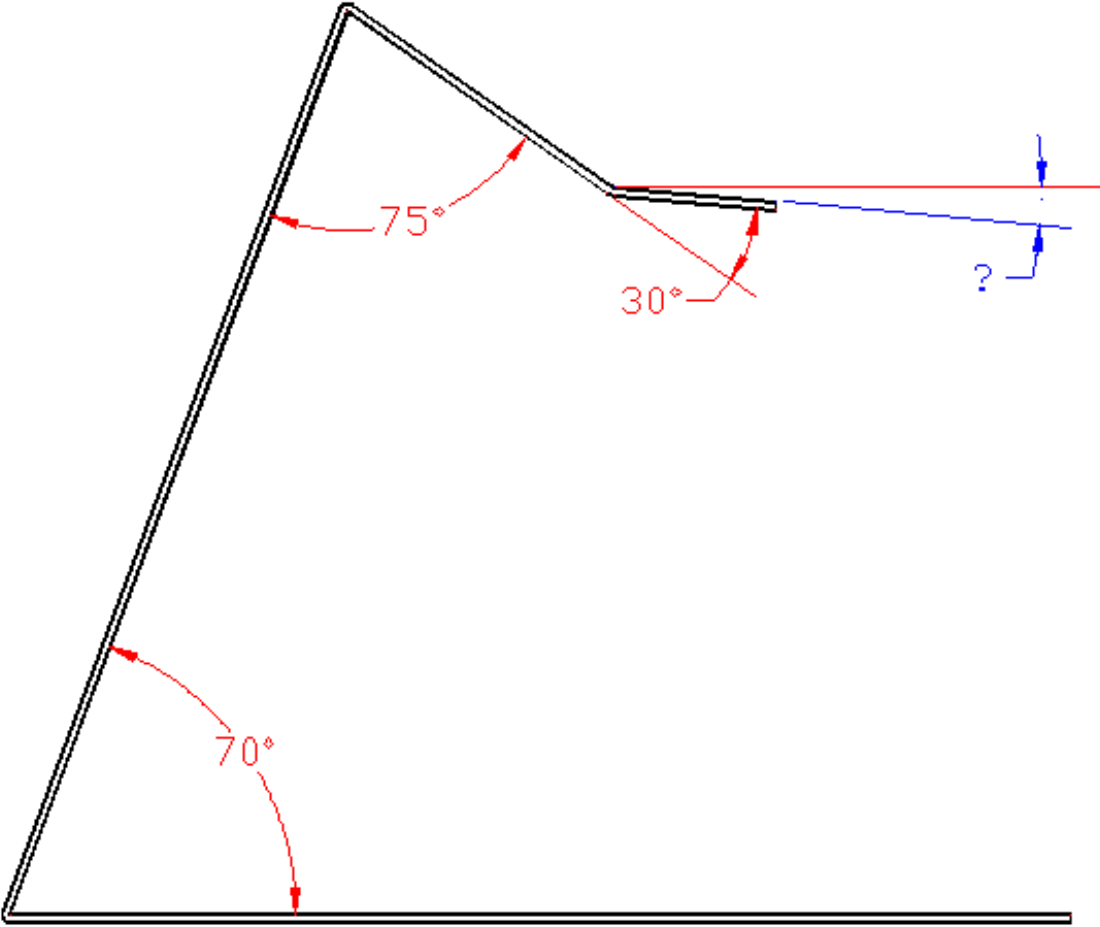
Do you see how to use angle facts #1 and #5 to prove that  $A+B+C= 180^\circ$ ?

## Worksheet 5-2: Finding Angles on Prints

Sometimes the angle given on a print cannot be measured directly. Then, you must use the angle facts and your knowledge of geometry to calculate the angle from other features that you can relate to the print dimension. Other times you may need to calculate an angle that is not given on the print.

The problems below are taken from industry prints of stampings. Calculate the missing angles. Note that all the angles are given from the horizontal.





## Worksheet 5-3: Angular Tolerances

We can measure angles more precisely than one degree. How do we break down a degree into smaller units?

There are three ways. Decimal degrees are commonly used and easy to work with. We could say “fifty-two and one-half degrees” and we could write it as “52.5°”. Decimal degrees usually have only one decimal place in manufacturing.

Secondly, partial degrees can be expressed as fractions, for example, 52½°.

The third way is with degrees, minutes and seconds.  
One circle = 360 degrees. One degree = 60 minutes. One minute = 60 seconds.

We could write this in math notation as:

$$1 \text{ revolution} = 360^\circ \qquad 1^\circ = 60' \qquad 1' = 60''$$

With degrees-minutes-seconds, we can specify an angle with great precision, for example: 23°47'32"

You would read this as 23 degrees, 47 minutes and 32 seconds. Be careful not to confuse minutes (') and seconds (") with feet and inches. We use the same symbols in both angular and linear dimensions.

Note that 1" = 1/3600 of a degree. This is a really small angle. Seconds of a degree are never used on piece part drawings. They are for navigation, astronomy, and land surveying. On a print you will only see degrees and minutes.

Regardless of how partial degrees are specified (decimals, fractions, or minutes) there is only one standard tolerance for angles given on a print. In other words, an angle of 45.50° is treated just like an angle of 45.5°. The number of decimal places doesn't matter and neither does it matter if the angle is given in degrees and minutes or in degrees and fractions of a degree.

The most common standard tolerances are:  $\pm 2^\circ$  or  $\pm 1/2^\circ$

If the angles that are on the part are mostly for clearance, the  $\pm 2^\circ$  will likely be the standard tolerance. If the angles are for mating surfaces, however, you are likely to see  $\pm 1/2^\circ$  as the standard tolerance. Special tolerances are also used for angles.

Note that  $\pm 1/2^\circ$  can also be written as a decimal ( $\pm 0.5^\circ$ ) or as 30 minutes ( $\pm 30'$ ).

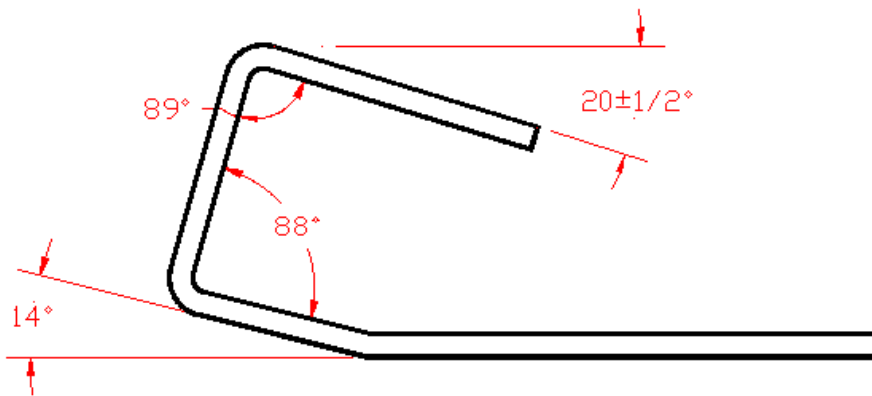
Remember: 30 minutes = 1/2 of a degree.

15 minutes = 1/4 of a degree.

Calculate the upper and lower limits for all the angular dimensions on the following three drawings. Write your answers next to each dimension on the drawings. Note that each drawing has its own tolerance block.

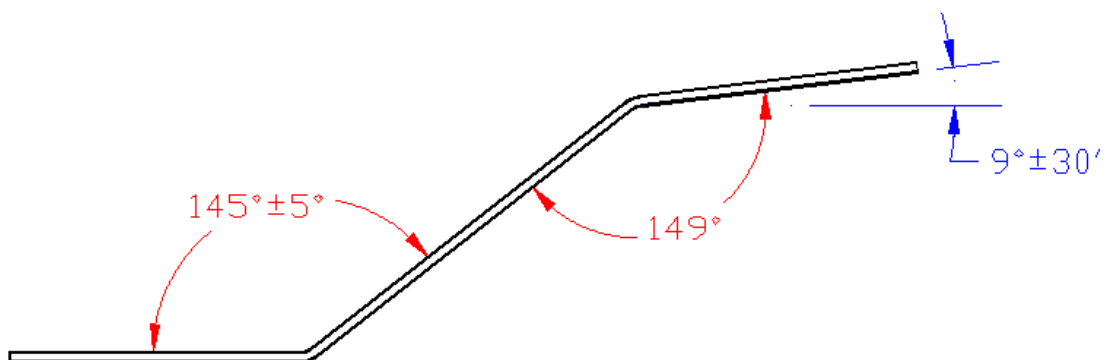
Unless otherwise specified, tolerances are:

- .XX        ±.01
- .XXX      ±.005
- Fractions ±1/32
- Angular   ±2°



Unless otherwise specified, tolerances are:

- .XX        ±.01
- .XXX      ±.005
- Fractions ±1/32
- Angular   ±2°

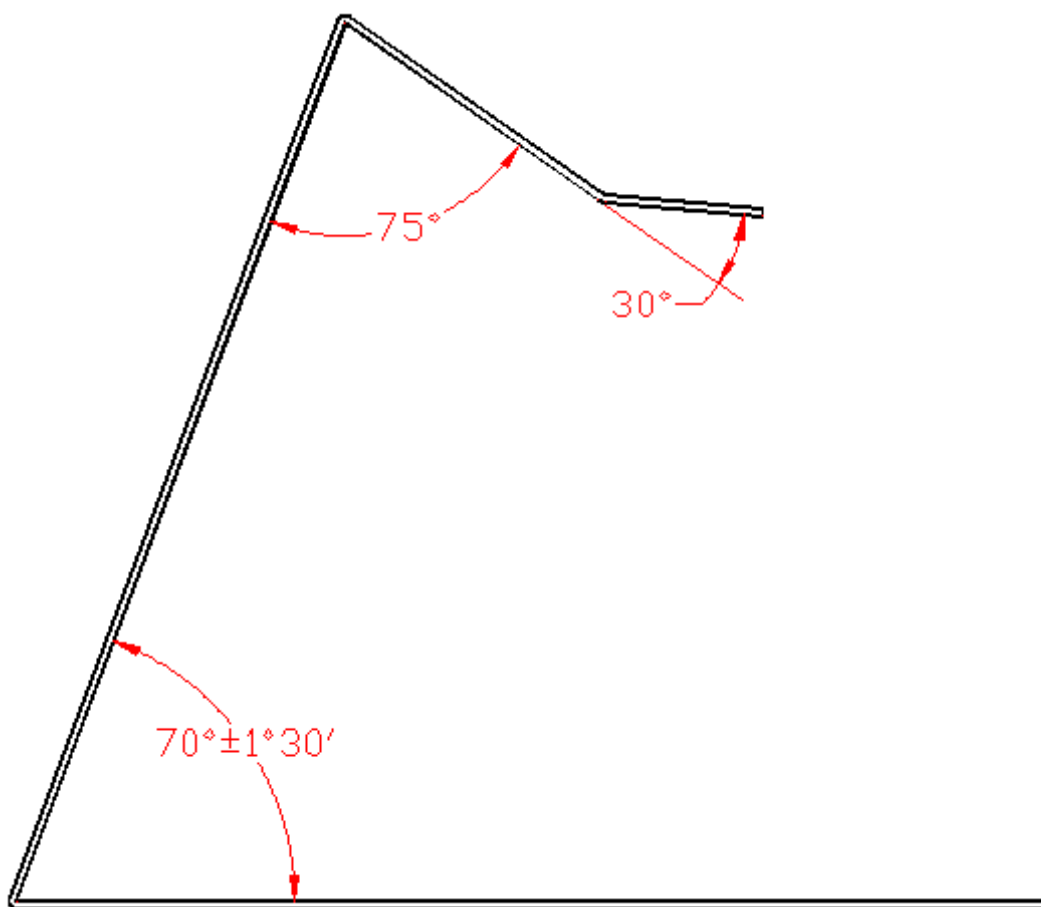


Unless otherwise specified, tolerances are:

.XXX ±.005

Fractions ±1/32

Angular ±30'



# Unit 6: Shop Problems

## MEET IT

Today in most manufacturing companies, problems are solved in teams. Quality control experts have learned that when workers cooperate to solve problems, the group's performance will be greater. In teams, workers brainstorm to propose new ideas and solutions. They learn to summarize and incorporate



the ideas of others to formulate the "best" solution to problems. Productivity certainly increases as workers on a team build trust and harmony and become more flexible.

To solve problems you often need to have additional information beyond what you are given. References such as machinist's handbooks and industry specification books can provide useful information to help you solve problems. You should know where these references are kept and which charts, tables, and formulas they contain. When you have all the information you need, you're ready to apply your math skills to come up with answers.

Your supervisor may ask you and several of your co-workers, as a team, to complete a specific project, which requires that you solve problems. To do this well, you have to communicate and work well with the team. On the job, good communication and teamwork are just as important as math skills. They will also make you more successful in your career.

## LEARN IT

In this unit you will learn to:

- Use formulas from industry handbooks
- Get the information you need from charts and tables
- Translate different units of measurement

## USE IT

- Tap drill size
- Tap drill clearance (for depth)
- Drill speeds



## WORKSHEET 6-1: Calculating Tap Drill Size

In order to machine a tapped hole, you need to leave enough stock to cut the threads. You can use this formula to select the proper drill size for a standard 60° tap.

Tap Drill Size, TDS = DIAMETER - 1/N

N refers to the number of threads per inch (tpi). Question : How many threads per inch do you have on a 5/16 - 18 thread? Answer: 5/16 is the diameter, you have 18 threads per inch.

Sample Problem 1: Find the tap drill size for a 3/8 - 16 tap.

$$\text{TDS} = \text{DIA} - 1/N$$

$$\text{TDS} = 3/8 - 1/16$$

$$\text{TDS} = .375 - .0625$$

$$\text{TDS} = .3125$$

$$.3125 = 5/16 \text{ drill}$$

Step 1 : Always write out your formula.

Step 2 : Plug in your variables.

Step 3 : Convert fractions to decimals.

Step 4 : Solve.

Step 5 : Use a 5/16 drill.

Sample Problem 2: Find the tap drill size for a 1/4 - 20 tap.

Step 1 : Always write out your formula.

Step 2 : Plug in your variables.

Step 3 : Convert fractions to decimals.

Step 4 : Solve.

Step 5 :  $13/64 = .203$

$$\text{TDS} = \text{DIA} - 1/N$$

$$\text{TDS} = 1/4 - 1/20$$

$$\text{TDS} = .250 - .050$$

$$\text{TDS} = .200$$

.200 is almost equal to  $13/64$ , so this would be the best drill.

TIP: This formula only works for cutting taps; it does not work for forming taps.

Practice Problems: Which drill should you use for each of the following tap sizes?

(1) 1/2 - 20

(2) 1/2 - 13

(3) 3/8 - 24

(4) 5/16 - 18

(5) 1/4 - 28

## Worksheet 6-2: Calculate Drill Clearance

In order to get the required length of thread, it is necessary to plunge the drill deeper than the depth specified on the print. This is because the drill is pointed (typically with a  $118^\circ$  angle) and we need to have the full diameter along the entire depth called out by the print. To calculate how far to plunge the drill, use this formula:

Drill Clearance,  $DC = \text{DIA}/3 + \text{Depth}$

Sample problem 3. Calculate Drill Clearance for a  $3/8$  DIA hole that is 1 inch deep.

$$DC = \text{DIA}/3 + \text{Depth}$$

$$3/8 = .375$$

$$DC = .375/3 + 1.000$$

$$DC = .125 + 1.000$$

$$DC = 1.125$$

Step 1 : Always write out your formula.

Step 2 : Convert fractions to decimals.

Step 3 : Plug in your variables.

Step 4 : Divide diameter by 3.

Step 5 : Add the depth.

Sample problem 4: Calculate Drill Clearance for a  $1/4$  Dia. x  $1 \frac{1}{2}$  inch deep hole.

$$DC = \text{DIA}/3 + \text{Depth}$$

$$1/4 = .250$$

$$DC = .250/3 + 1.500$$

$$DC = .083 + 1.500$$

$$DC = 1.583$$

Step 1 : Always write out your formula.

Step 2 : Convert fractions to decimals.

Step 3 : Plug in your variables.

Step 4 : Divide diameter by 3.

Step 5 : Add the depth.

Practice problems. Find clearance for the following holes.

(6)  $1/2$  DIA x 1 in.

(7)  $1/4$  DIA x 2 in.

(8)  $3/8$  DIA x  $1 \frac{1}{2}$  in.

(9)  $5/16$  DIA x  $\frac{1}{2}$  in.

(10)  $17/32$  DIA x  $\frac{1}{2}$  in.

## Worksheet 6-3: Calculating Drill Speeds in Revolutions Per Minute (RPM)

It is very important to set the spindle speed at the correct RPM. Using the wrong RPM places undue stress on both the tool and the machine. Unfavorable cutting conditions may result in poor quality, worn or broken tools, etc.

The two main factors are:

- (1) What material are you cutting?
- (2) What is the diameter of the drill?

The harder the material, the lower the RPM. The larger the drill, the lower the RPM. Of course, RPM increases with soft material or small diameter drills.

In order to calculate RPM, you can use this formula:

$$\text{RPM} = 4 \times \text{CS/DIA}$$

In this equation, CS refers to the material cutting speed indicated on the chart below.

### Materials Chart

Material	Cutting Speed
Wood	350 ft / min
Brass	250 ft / min
Aluminum	200 ft / min
Plastic	125 ft / min
Cast Iron	100 ft / min
Machine Steel	80 ft / min
Tool Steel	60 ft / min
Stainless Steel	40 ft / min
Marble	20 ft / min

Sample Problem 5: Find the correct RPM for cutting tool steel with a 5/16 in. drill.

$$\text{RPM} = 4 \text{ CS/DIA}$$

$$\text{CS} = 60 ; 5/16 = .312$$

$$\text{RPM} = 4 (60)/.312$$

$$\text{RPM} = 240/.312$$

$$\text{RPM} = 769$$

Step 1 : Always write out your formula.

Step 2 : Select the correct CS; convert the fraction.

Step 3 : Plug in your variables.

Step 4 : Multiply. (4 x 60)

Step 5 : Divide. (240 ÷ .312)

Sample Problem 6: Find the correct RPM for cutting aluminum with a 13/64 in. drill.

$$\text{RPM} = 4 \text{ CS/DIA}$$

$$\text{CS} = 200 ; 13/64 = .203$$

$$\text{RPM} = 4 (200)/.203$$

$$\text{RPM} = 800/.203$$

$$\text{RPM} = 3940$$

Step 1 : Always write out your formula.

Step 2 : Select the correct CS; convert the fraction.

Step 3 : Plug in your variables.

Step 4 : Multiply. ( $4 \times 200$ )

Step 5 : Divide. ( $800 \div .203$ )

Compare the answers in these two problems. Notice how the material conditions make a big difference.

TIP: Of course most machines do not have a full range of spindle speed settings. For example, instead of setting the spindle speed at 769, you may have to choose between 600 and 800 RPM. Remember that you would choose the lower setting when operating an older machine.

Practice Problems. Calculate RPM for the following set-ups.

(11) 1/2 drill; stainless steel

(12) 1/2 drill; brass

(13) 3/8 drill; plastic

(14) 5/16 drill; aluminum

(15) 1/4 drill; cast iron

# Unit 7: Beyond Arithmetic

## MEET IT

Whether you realize it or not, you've met math in many places and will continue to run into math throughout your life. However, there's more to math than arithmetic -- adding, subtracting, multiplying, and dividing. You will need additional math skills not only on the shop floor but also when you buy a house, prepare a monthly budget, calculate your car's gas mileage, figure out how much tile to buy for the bathroom, see how well your investments did, or complete your tax return. You will regularly use problem-solving, critical thinking, estimation, and calculator skills. Challenge yourself!



## LEARN IT

In this unit you will learn how to:

- Create your own algebraic formulas for shop tasks on the job
- Calculate monthly payments for auto loans and mortgages on a scientific calculator
- Use signed numbers
- Use Cartesian coordinates to plot number pairs

## USE IT

- Creating your own algebraic formulas for shop tasks
- Did you get cheated on your car loan?
- Signed numbers and rectangular coordinates

## WORKSHEET 7-1: Creating Shop Formulas

If you have to perform similar calculations many times you can use a formula to save time and effort. Using formulas means replacing specific numbers with letters that represent any number. This is moving us into algebra.

When you have to make a calculation, you replace the letters in the formula with the actual numbers that you need for the calculation. The letters are called “variables” because the actual numbers vary from problem to problem.

You used formulas to solve shop problems in Unit 6. For example:

Tap Drill Clearance = (Tap Drill Diameter)/3 + Depth

We can write this formula more like an algebraic equation, using the following letters for variables:

C = clearance, includes the extra depth we need

Ø = the diameter of the tap drill

D = the print dimension for the depth of the hole

$$C = \frac{\text{Ø}}{3} + D$$

So, for a 7/8 drill to make a hole that has the full 7/8” diameter all the way down to a 2” depth, we plug in our values and solve:

$$\frac{.875}{3} + 2 = 2.2917$$

But what if the hole is threaded? The print will call out the thread specification but not the tap drill size. How deep do you drill for a 7/8-14 thread 1” deep? You will have to find the tap drill size and then calculate the clearance.

The tap drill size is  $\frac{7}{8} - \frac{1}{14} = .875 - .071 = .804$  Now plug this into the formula:

$$C = \frac{.804}{3} + 1.000 = .268 + 1.000 = 1.268$$

Now you will make your own formulas.

Make a Shop Formula :

You are operating a band saw and you have to cut bar stock into blanks for the “chucker.” (A chucker is a production lathe that holds the workpiece in a chuck. It has to be loaded and unloaded one piece at a time. )

Make a formula that will calculate the minimum length for the starting pieces.

- Overall length of the part (OAL)
- Tolerance for the OAL
- The width of the saw blade
- The “Clean Up”, that’s the length of material you have to remove to get rid of the saw cut marks.
- Holding Length, the material the chucker needs to grab to hold the part securely

A) What are your variables?

B) What is the formula?

Now use your formula to solve the following problems:

1) The print dimension for the OAL is  $3.025 \pm .015$ ; the saw is  $1/16$ " wide. The chuck needs to grab  $3/4$ " of material for a solid hold and you also need to leave at least  $.045$  to clean up the saw grooves. How long should the blanks be for a chucker operation where you flip the part over to cut both front and back sides?

2) What is the blank size if the part OAL becomes:  $2.860^{+.020}_{-.005}$

3) Same part as #2, same chucker, but saw blade is now  $.045$  wide and finer toothed. You can clean up by facing off only  $.025$  on each side. What is the size of the blank?



4) A bar-fed lathe is the alternative to a chucker. It runs continuously, feeding out the stock for each piece and cutting off the part when it's finished.

A) Write a formula to calculate how many pieces you should get from a 12-foot piece of bar stock. Account for the print dimension for the OAL, the width of the cutoff tool, and the length of the "bar ends" or the amount of material wasted because the bar feed has to have something to hold onto.

B) Use this formula to calculate the maximum number of pieces you can get from a 12 foot bar for a part with an OAL of  $1.892 \pm .010$ , a cutoff tool .110 wide and a bar end of 18".

C) The same part but cutoff tool is now .065 wide. How many pieces per 12 ft. bar?

## WORKSHEET 7-2: Compound Interest and Monthly Payments

If you borrow money to buy a car or a house you will have to make monthly payments. When you are shopping for a car, for example, the salesman will often do some quick work with a calculator to tell you what your monthly car payments would be. Sometimes the car salesman will ask, "How much do you want to spend on car payments?" He will then work backwards and figure out how much car you can buy for that monthly payment.

These monthly payments are based on the amount of money that you borrow, the interest rate on the loan, how long you will have to pay it back, and how many payments. Since they involve compound interest, they are not easy to calculate. Solving these kinds of problems requires using complex formulas and numbers with negative exponents.

Unfortunately, dishonest people may take advantage of the fact that most people are not fluent with algebra. Sharks will try to charge higher monthly payments than are allowed for a particular interest rate. Even though this is illegal, it's hard to catch them, because the calculations are hard and few people challenge them.

Explaining all the math involved in compound interest is beyond the scope of this book but that's OK. If you can learn to use a scientific calculator, you can still protect yourself from sharks. In fact, most real estate agents and car salesman don't understand all the math either. They use special calculators that are designed for people to do financial calculations whether they are good at math or not.

(Of course, being able to use the calculator does not replace the need to learn algebra. It's always better to know all the math than just a formula!)

The formula we use to figure out the regular payments for compound interest may look scary but you can work it out with a TI-30 or another scientific calculator.

$$\text{Payment} = \frac{\text{Payment} \times \text{Interest}}{1 - (1 + \text{Interest})^{-n}}$$

The Payment is usually a monthly payment.

The Principal is the amount you borrow.

"n" is the total number of payments you would have to make.

The Interest is a rate, expressed as a percent that you pay on the principal each month.

For this formula the interest is based on the Annual Percentage Rate (or APR) of the loan. Look at car ads or at bank ads in the real estate pages of the newspaper. They tell you what APR they charge for loans. The APR is the rate when you consider the whole year. We need to figure the interest rate for each payment period, which happens to be a month. So,

$$\text{Interest} = \text{Annual Percentage Rate}/12$$

We will use this formula to figure what our monthly payments would be for a \$10,000 auto loan for 3 years with an APR of 9.5%

Principal = \$10,000, what we want to borrow.

n = number of monthly payments, for 3 years that is  $3 \times 12 = 36$  periods.

Interest = if the APR is 9.5%, (which is .095 when expressed as a decimal), the rate we need for the calculation is:

$$.095 \div 12 = .00792 \quad (\text{rounded off to 5 decimal places}).$$

Now plug these numbers into our formula:

$$\text{Payment} = \frac{10,000 \times .00792}{1 - (1 + .00792)^{-36}} = \frac{79.167}{1 - (1.00792)^{-36}}$$

But sometimes we get stuck with that -36 exponent!

This  $(1.00792)^{-36}$  actually means: "raise the number 1.00792 to a power of 36 and then divide that number into 1." How are we going to do that?

With  $\boxed{y^x}$  the key, that's how. Find this key on your calculator. You will also need the parentheses keys:  $\boxed{(}$  and  $\boxed{)}$

Here's how you can do it on a TI-30 calculator. The keystrokes are:

$$10,000 \boxed{\times} .00792 \boxed{\div} \boxed{(} 1 \boxed{-} \boxed{(} 1 \boxed{+} .00792 \boxed{)} \boxed{y^x} 36 \boxed{+/-} \boxed{)} \boxed{=}$$

(Note that other scientific calculators should have similar keys.)

Your answer should be: **\$320.35**

Solve problems using this keystroke sequence on the TI-30a calculator:

Principal  $\times$  Interest  $\div$  ( ( 1 - ( ( 1 + Interest )  $y^x$  n +/- ) =

Calculate what the monthly payments would be for the following loans. Remember the Interest has to be calculated from the APR :

$$\text{Interest} = \frac{\text{APR}}{12}$$

1) A 5 year car loan of \$25,000 at 6.5% APR.

2) A 5 year car loan of \$25,000 at 1.9% APR.  
(Note: when you go to buy a car, bring this formula and your calculator!)

Extra: If you decide to buy this \$25,000 car and the dealer offers you a choice between:  
 a low finance rate of 1.9% APR  
 or \$1500 cash back with a finance rate of 6.5%,  
 Which offer would you take? (This is a hard choice!)

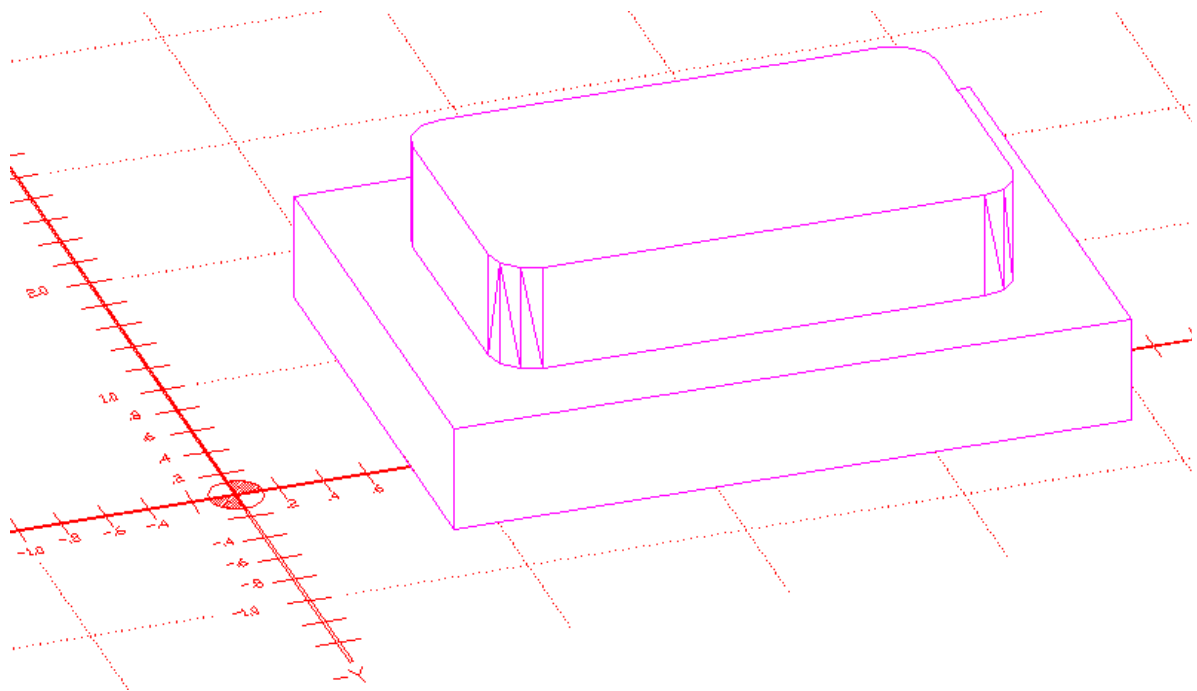
3) What are the monthly payments on a 30-year mortgage for \$127,000 at 7.75% APR?

## WORKSHEET 7-3: Signed Numbers and Rectangular Coordinates

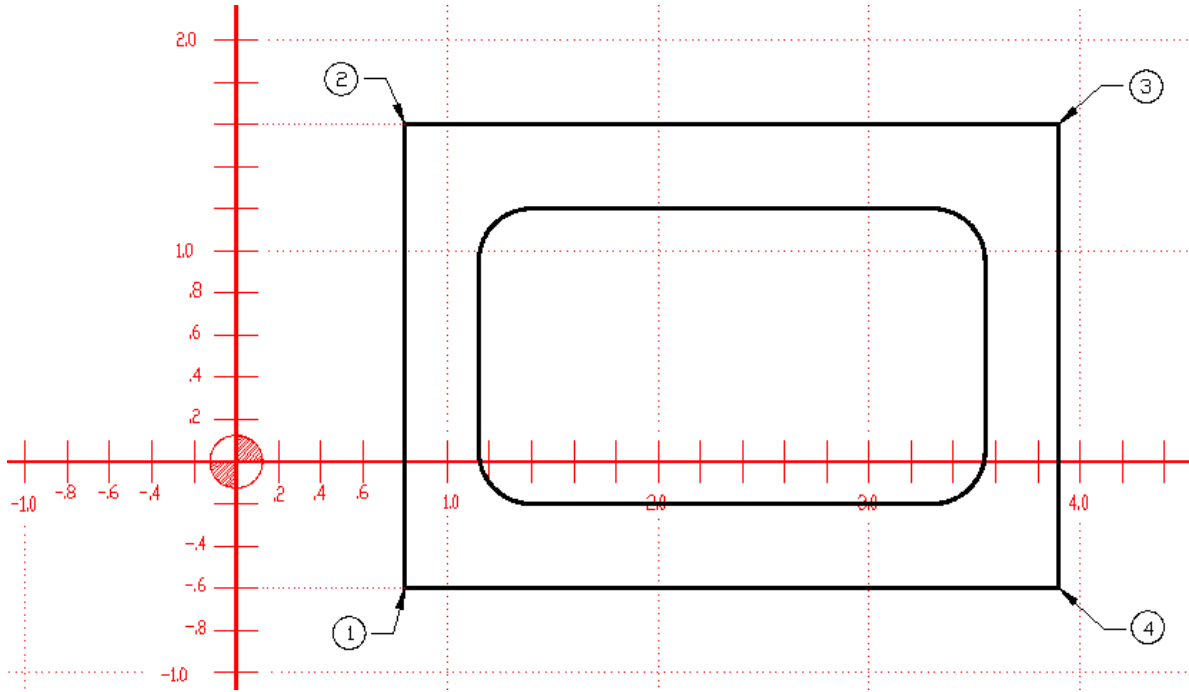
In CNC programming we must use both positive and negative numbers. When we tell the tool to move we have to tell it how far and in what direction. We use positive and negative numbers to indicate direction.

The CNC machines move from point to point. To program them we give the location of the next point as a coordinate in a reference system. Each line of the CNC program calls out a new point and the tool moves right to it. The reference system we use has an X and a Y-axis. Each point is called out as a pair of numbers with the X value first and the Y second. So, point 2,3 is a point that lines up with the +2 on the X-axis and the +3 on the Y-axis. The point 2,-5 would line up with +2 on the X and -5 on the Y-axis.

If you are going to write a program to mill around the edge of a block, you will have to call out the X and Y coordinates for each corner in a clockwise direction. The cutting tool will then move in straight line along each edge.



Look at the drawing below. This is the top view of the block on the previous page.



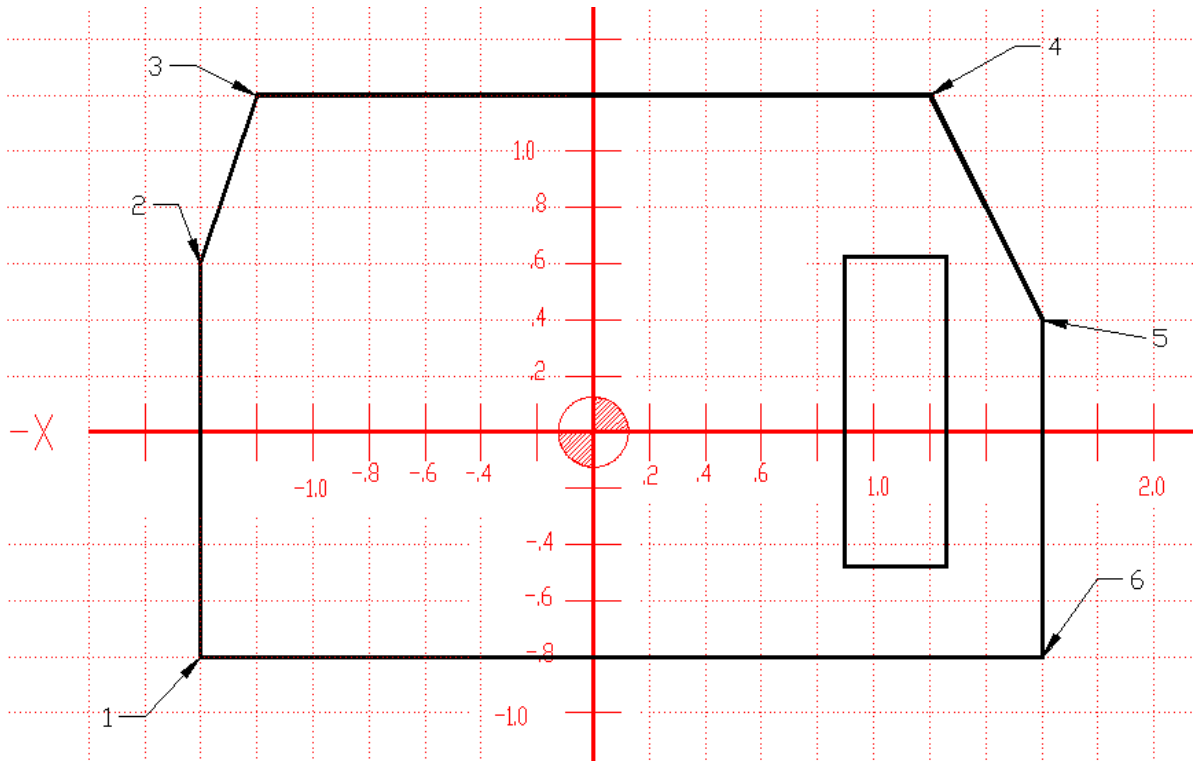
What are the coordinates for each corner?

- 1) .8, -.6
- 2) \_\_\_\_\_
- 3) \_\_\_\_\_
- 4) \_\_\_\_\_

These X,Y coordinates are used all the time in algebra and mathematicians call them Cartesian coordinates. In machine shops they are often just called “X,Y” coordinates.

In math, Cartesian coordinates are written as a pair of numbers with the X coordinate first and the Y coordinate second, for example: 2.865, 3.091. However, in CNC programs coordinates are written with the letter that identifies which coordinate it is. So the coordinates in the previous example would be written as: X2.865 Y3.091. Furthermore, they don’t have to be written in any particular order since the letter tells the machine tool which number goes with which axis.

Look at the drawing below and enter the coordinates for all the points indicated. Use the CNC programmer's practice of putting the Y or X before the coordinate:



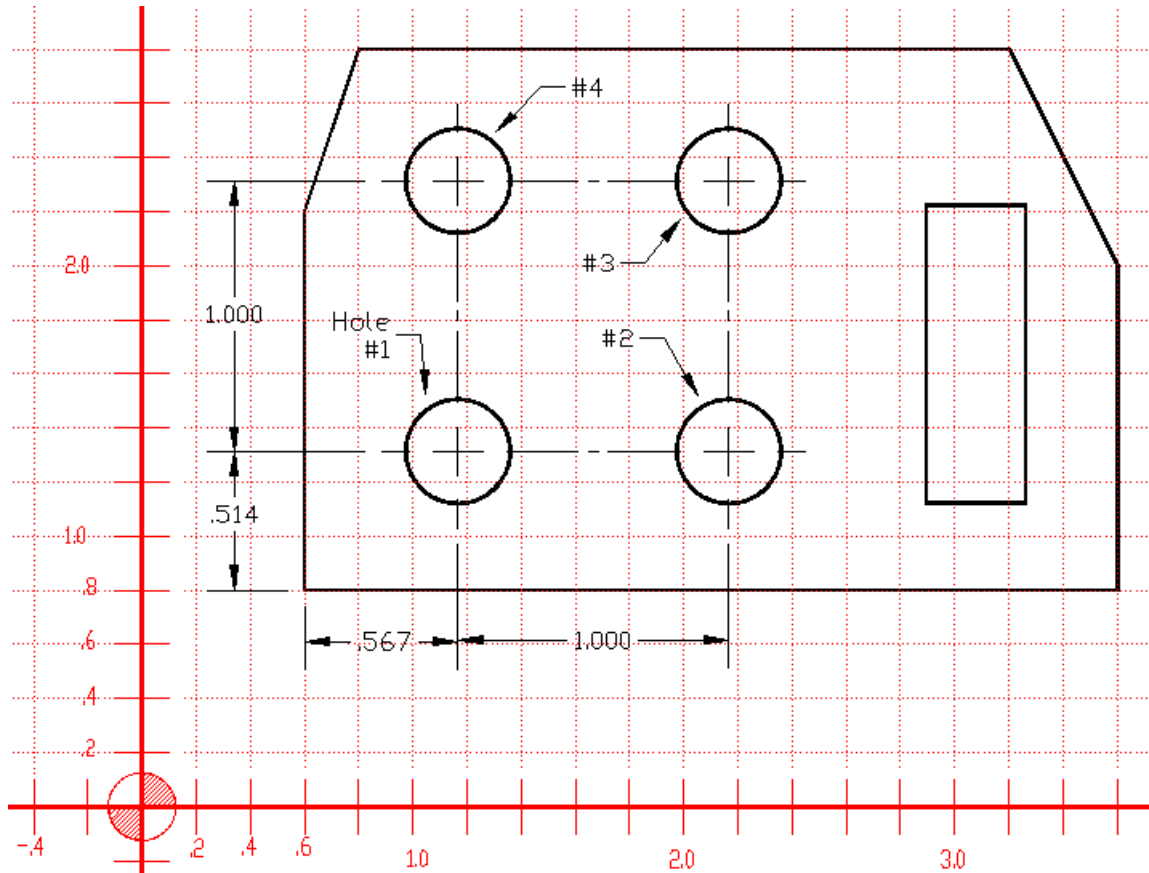
- 1) \_\_\_\_\_
- 2) \_\_\_\_\_
- 3) \_\_\_\_\_
- 4) \_\_\_\_\_
- 5) \_\_\_\_\_
- 6) \_\_\_\_\_

Incremental Moves:

The examples above are based on coordinates that are locations on a reference system. The point  $-0.8, 0.6$  is an absolute coordinate and when the program calls: "X $-0.8$  Y $0.6$ " the tool goes straight to that point from wherever it happens to be when the command is issued. Most CNC programming is done with absolute coordinates.

The alternative method is to program with incremental moves. This is when the program tells the tool to move a specific distance instead of telling the tool to go to a specific point.

While it is generally better to program with absolute coordinates, sometimes it is easier to specify a move as an increment. The print below is an example:



The corner of this part is located at X.6 Y.8 so the coordinate for the location of the center of hole #1 is:  
 $X = .6 + .567 = 1.167$   
 $Y = .8 + .514 = 1.314$

or X1.167 Y1.314. The CNC machine would move the drill to this spot and drill the holes in the order 1, 2, 3, 4.

What would the incremental moves be? From hole 1 to hole 2:  
 $X=1, Y=0$ , so the incremental move is written as: X1 Y0

1) What is the incremental move it from Hole 2 to Hole 3? \_\_\_\_\_

2) From Hole 3 to Hole 4? \_\_\_\_\_

3) What is it from Hole 4 back to Hole 1? \_\_\_\_\_



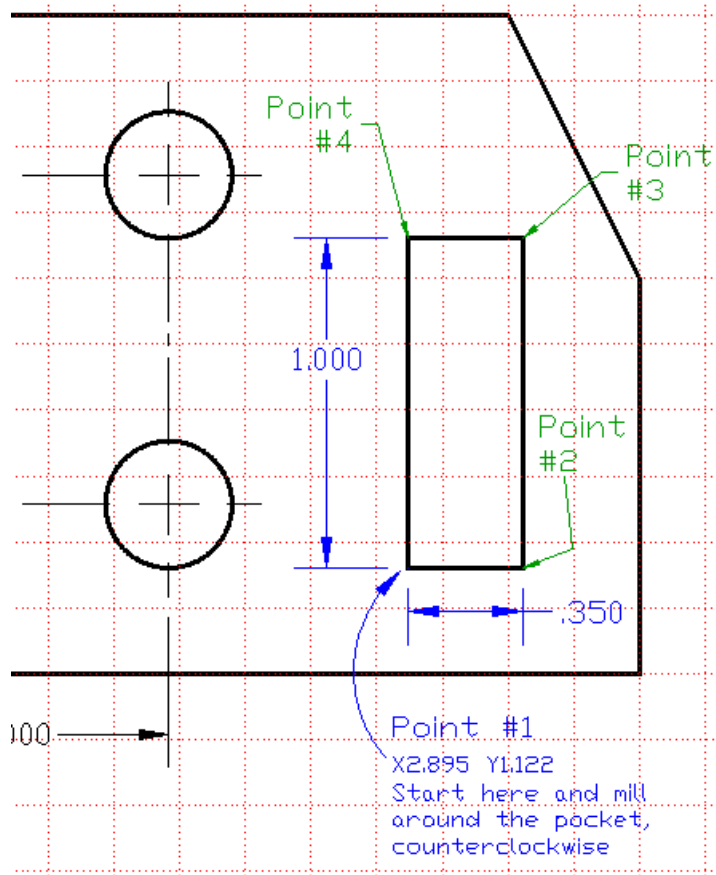
It probably seems a little easier to write the relative coordinates for incremental moves than to write out the absolute coordinates because the holes are spaced with nice round numbers. However, that is not the reason for using incremental moves.

What if we had to change the location of the hole pattern? This happens frequently with design changes. The hole size and spacing stays the same but the holes as a group get moved to a new location.

If we had programmed it in absolute coordinates we would have to change the coordinates for each hole. If we used incremental moves, however, we would only have to change one line of code, the line that gives the location for the first hole drilled.

It is usually best to use absolute coordinates in CNC programming, but sometimes it is better to use incremental (or relative) programming on features that will likely be changed later on. Skilled programmers think ahead and prepare for the inevitable design changes.

Now look at the rectangular pocket on the right side of the drawing. The lower left corner of the pocket is X2.895 Y1.122.



Give the incremental moves from:

Point #1 to #2: \_\_\_\_\_

Point #2 to #3: \_\_\_\_\_

Point #3 to #4: \_\_\_\_\_

Point #4 to #1: \_\_\_\_\_

# Unit 8: Triangles and the Pythagorean Theorem

## MEET IT

People have been fascinated by the connection between shapes and numbers for thousands of years. The measurements of squares, circles, rectangles, and triangles had seemingly mystical relationships that only a very few people understood. Today, anyone can learn in school the secrets of the greatest philosophers of ancient times.



One of the most famous philosophers was Pythagoras of ancient Greece. We might call him an early mathematician, but the concept of mathematics hadn't been invented yet! Pythagoras figured out a special relationship concerning the lengths of the sides of a right triangle, that is, a triangle in which one of the angles is a right angle. We call this relationship the "Pythagorean theorem" in honor of his discovery. It's probably the most useful concept in all of geometry.

## LEARN IT

In this unit you will learn to:

- Work with squares and square roots
- Identify special types of right triangles: 45 degree, 3-4-5, 30-60-90
- Use the Pythagorean theorem to find the sides of right triangles
- Use the Pythagorean theorem to find missing print dimensions

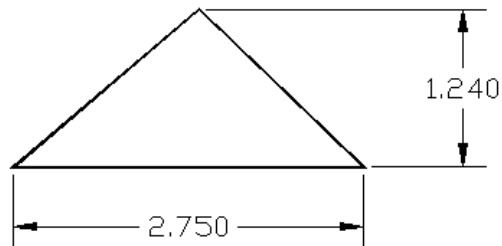
## USE IT

- Finding areas of triangles
- Calculate lengths using the Pythagorean theorem
- Working with triangle types: 45-degree, 30-60-90-degree
- Finding minimum diameter of bar stock to make blanks for milling
- Finding missing dimensions on slotted holes

## LEARN IT

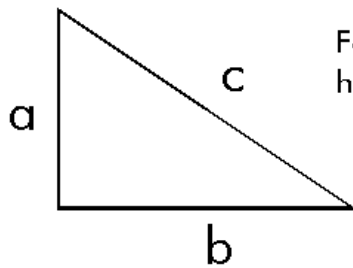
Triangle Fact #2:

$$\text{AREA OF A TRIANGLE} = \frac{1}{2} \times \text{BASE} \times \text{HEIGHT}$$



So for this triangle  
the area is  
 $\frac{1}{2}(2.75)(1.24) =$   
1.705

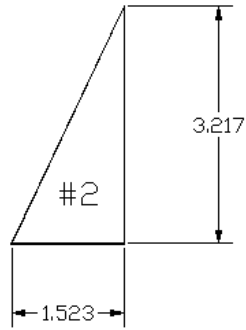
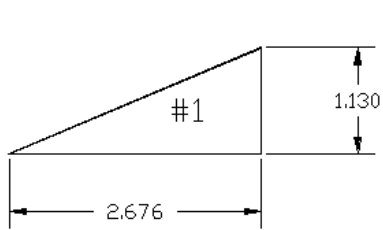
This is an easy formula but often the height is not given. However, in a machine shop we deal mostly with right triangles and that makes this calculation simpler.



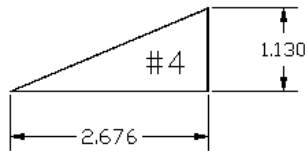
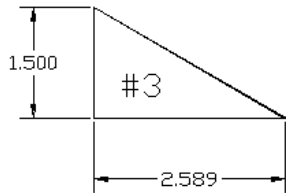
For right triangles, the base and height are the legs of the triangle.

### Worksheet 8-1: Areas of Right Triangles

Find the areas of the four triangles below:



- 1. -----
- 2. -----
- 3. -----
- 4. -----



## LEARN IT: Triangles and Right Triangles

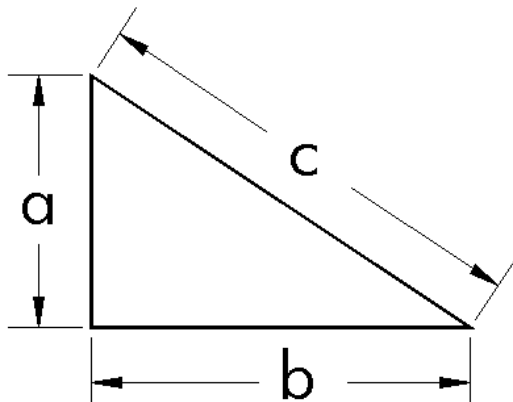
An angle of  $90^\circ$  is called a right angle. Any triangle that has a  $90^\circ$  angle in it is a right triangle. For metalworking, right triangles are the most important kind of triangle.

There is an important mathematical relationship that is true for all right triangles. It is called the Pythagorean theorem. It is used in many ways in shop math. It is also important for engineering and science.

Pythagorean theorem:

For any right triangle, the length of its longest side will equal the square root of the sum of the squares of the other two sides.

Look at the triangle below. The length of each side is represented by a letter: a, b or c. Side "c" is the longest side and is called the hypotenuse.



In math notation we write the Pythagorean theorem as:

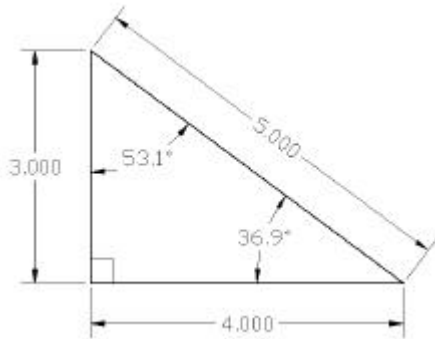
$$c^2 = \sqrt{a^2 + b^2}$$

We can also re-arrange this equation to get a new equation that we can use to find one of the short sides:

$$a^2 = \sqrt{c^2 - b^2}$$

So if we know just 2 of the three sides of a right triangle, we can always find the length of the third side.

You can test this by drawing a right triangle with a ruler. Make the sides 3" and 4" long.



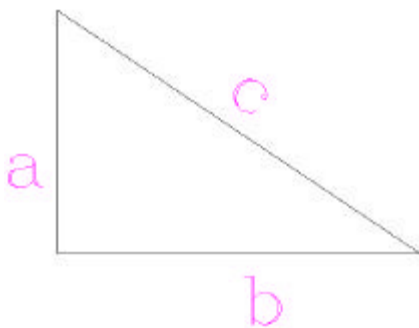
If you work out the formula, you get:

$$\sqrt{a^2 + b^2} = \sqrt{(3 \times 3) + (4 \times 4)} = \sqrt{9 + 16} = \sqrt{25} = 5$$

For a right triangle with 3" and 4" sides, the hypotenuse is 5". This is called a 3-4-5 triangle and it is very useful. Among other things, we can use it to layout the walls in a building and make sure they are square to each other.

It works for a 3-4-5 triangle, but how do we know that the Pythagorean theorem will work for all triangles?

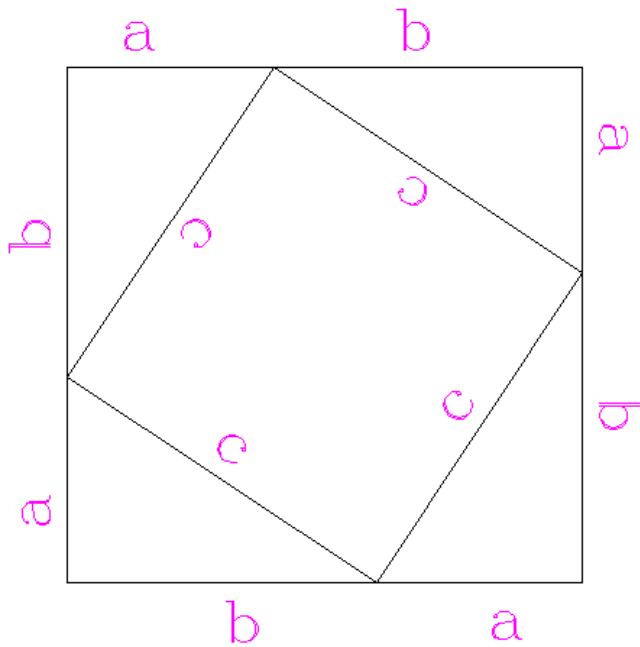
Suppose we have a right triangle with sides of lengths a, b and c:



Remember that the area of a triangle is  $\frac{1}{2}$  the base times the height, or

$$\text{Area} = \frac{1}{2}(a)(b)$$

Now take four of these triangles and make a large square out of them. The hypotenuses will line up to form a smaller square in the center:



The area of the big square is the length of one side squared, or

$$(a+b) \times (a+b)$$

Multiplying this out gives us:

$$a^2 + 2ab + b^2$$

We can also calculate the area for the big square by adding up the area of all of its component shapes, the four right triangles and the small square in the center. We then get:

$$\text{Total Area} = 4 \times \left(\frac{1}{2} ab\right) + c^2 = 2ab + c^2$$

Now we have two different equations to describe the area of the big square. Since they both describe the same area, we can write them as an equation:

$$2ab + c^2 = a^2 + 2ab + b^2$$

Simplify by subtracting  $2ab$  from each side and we get:

$$c^2 = a^2 + b^2$$

Now take the square root of each side:

$$\sqrt{c^2} = \sqrt{a^2 + b^2}$$

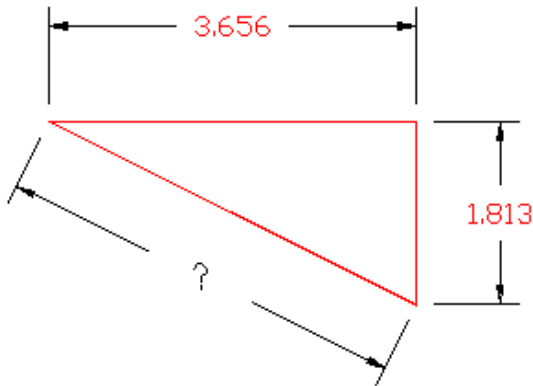
However, the square root of  $c^2$  is just  $c$ , so we now have:

$$c = \sqrt{a^2 + b^2}$$

This is the Pythagorean theorem!

## Worksheet 8-2: Pythagorean Theorem

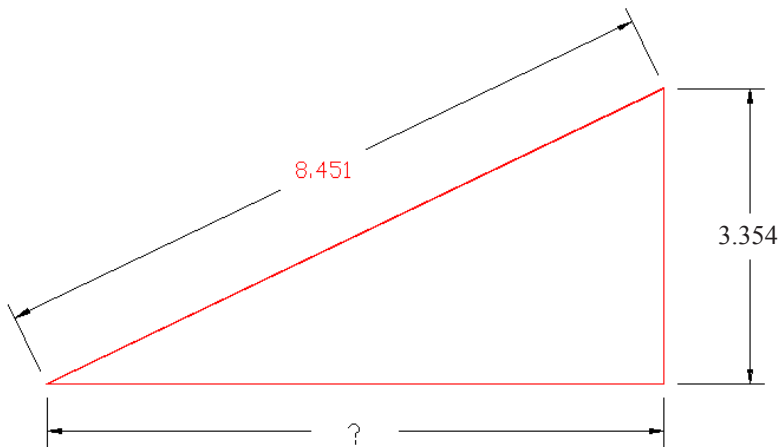
Examples:



Using the Pythagorean theorem and a calculator we can find the length of the hypotenuse. Round your intermediate answers to four places and round off your final answer to 3 places.

$$(3.656)^2 + (1.813)^2 = 13.3663 + 3.2870 = 16.6533$$

The square root of 16.6533 is 4.081 and that is the length of the hypotenuse.

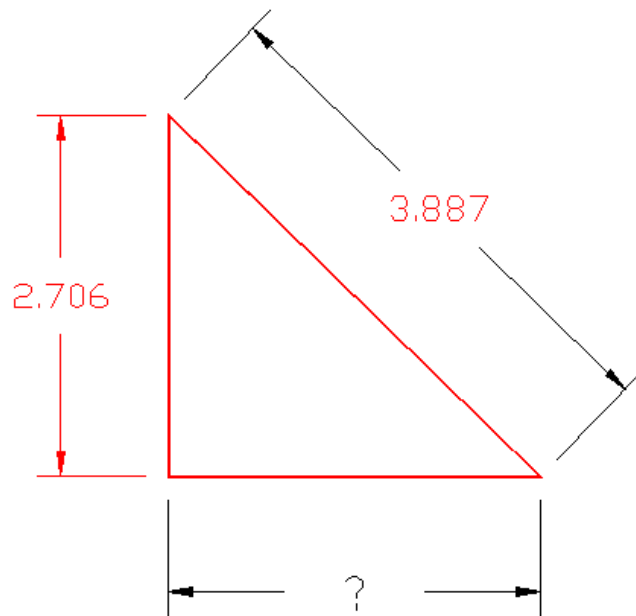
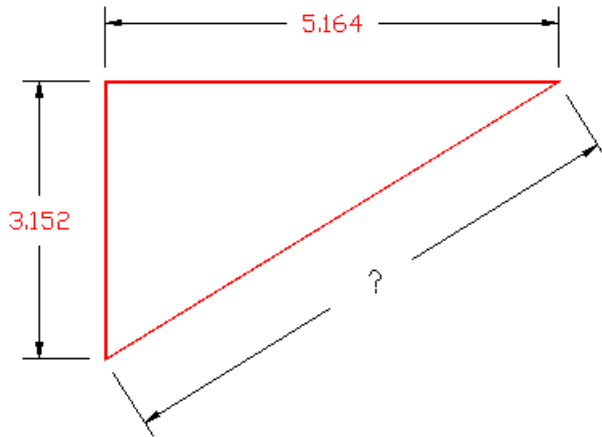


This example gives the hypotenuse and asks for the length of the side. Now you use the other formula.

$$a^2 = \sqrt{c^2 - b^2} \text{ which in this case gives us: } \sqrt{8.451^2 - 3.354^2} = \sqrt{71.4194 - 11.2493} = \sqrt{60.1701} = 7.757 \text{ (rounded to 3 places)}$$



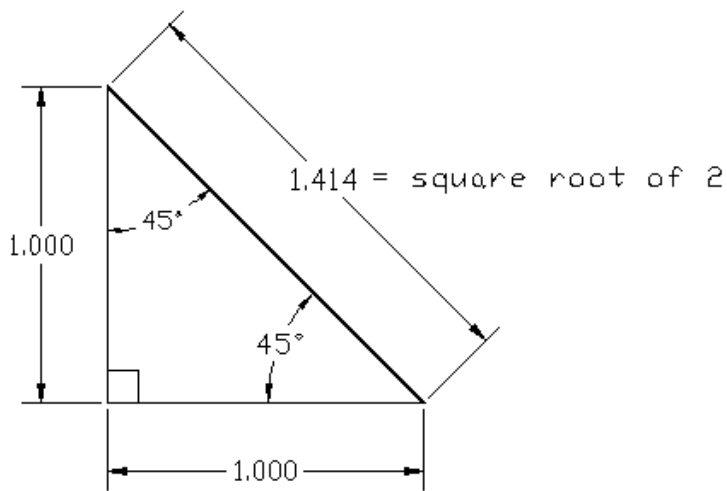
Use the Pythagorean theorem to find the length of the missing side on the triangles below.  
Give your answer to 3 decimal places.



## LEARN IT: SPECIAL TRIANGLES

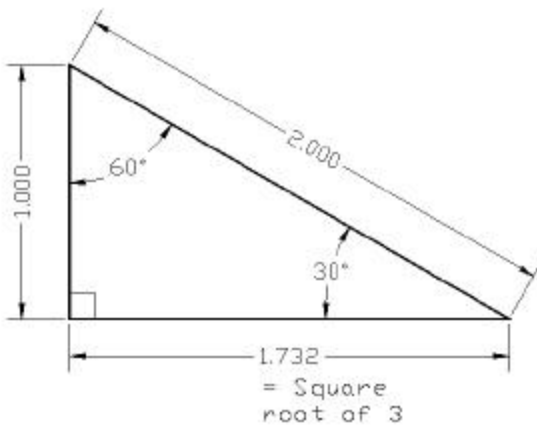
There are 3 special triangles that we run into frequently.

- 1) The 3-4-5 triangle which is shown earlier in this unit.
- 2) Right isosceles triangle, or the 45° right triangle.



Multiply the length of the side by the square root of 2 (which is 1.414) and you get the hypotenuse.

- 3) The 30-60-90 Triangle.

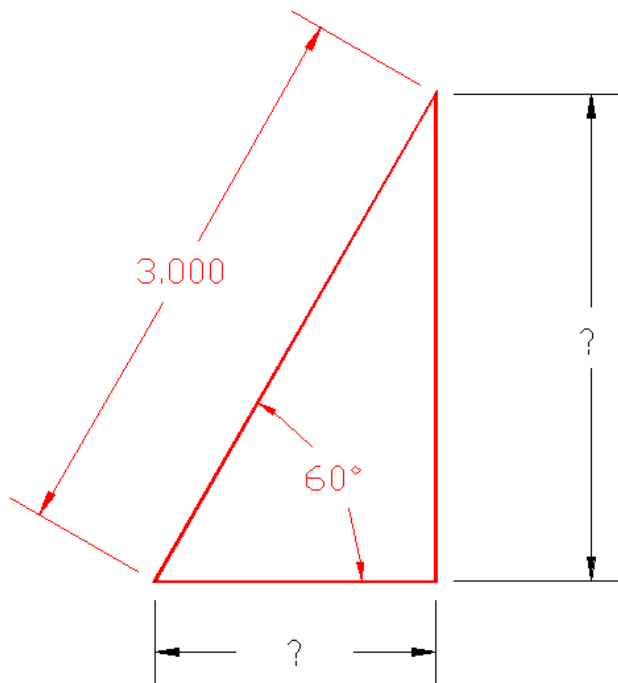
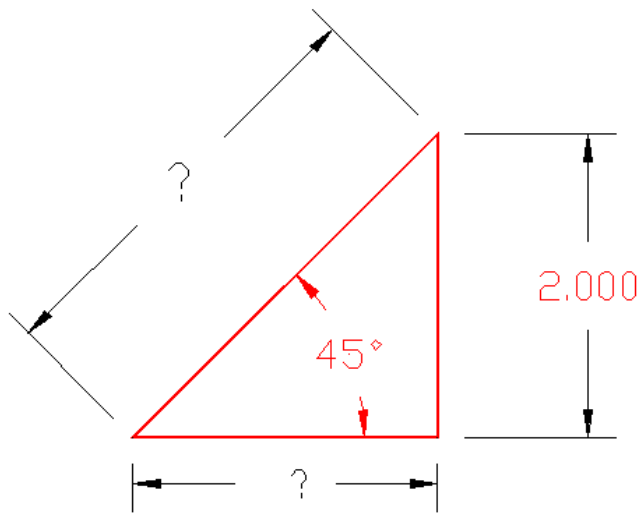


Multiply the short side by the square root of 3 (1.732) and you get the length of the long side.

Double the length of the short side and you get the length of the hypotenuse.

## WORKSHEET 8-3: 45° and 30-60-90 Triangles

Solve the triangles below. Use the special triangles to find the lengths of the missing sides.



## Worksheet 8-4: Sawing Blanks for Milling from Round Stock

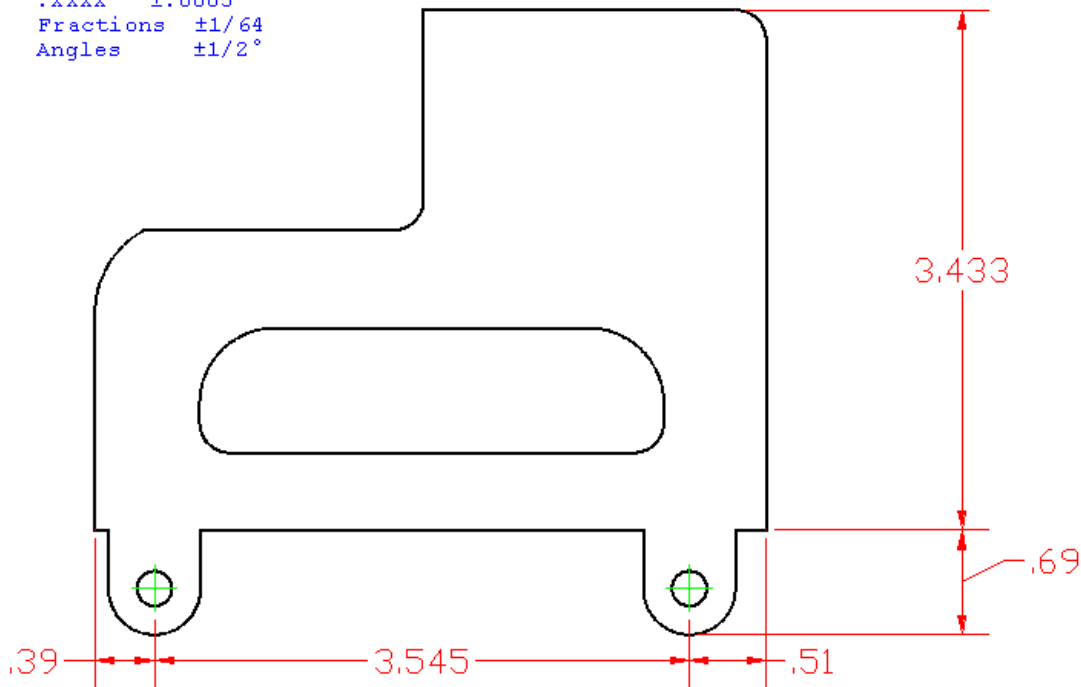
Sometimes irregularly shaped parts have to be milled from round pieces of stock. Since bar stock is readily available, shops may keep a wide range of bars on hand not just for lathe work but also for use as stock on short-run milling jobs.

The saw operator may only get a print of the final part with the job order. He or she would then have to use the Pythagorean theorem to figure out what diameter of bar stock should be used to cut the blanks.

Example:

Look at the print below. The first step for solving this problem is to find the overall dimensions.

Unless otherwise specified tolerances are:  
 .XX ±.01  
 .XXX ±.005  
 .XXXX ±.0005  
 Fractions ±1/64  
 Angles ±1/2°

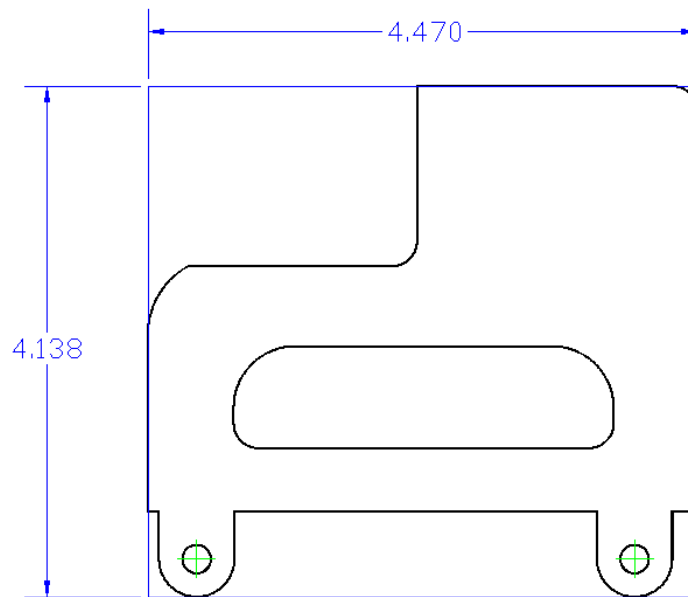


To get the overall length we not only have to add  $.39 + 3.545 + .51$ , we also have to account for the tolerances. So, what we should add is not the nominal dimensions but the upper limit of each dimension.

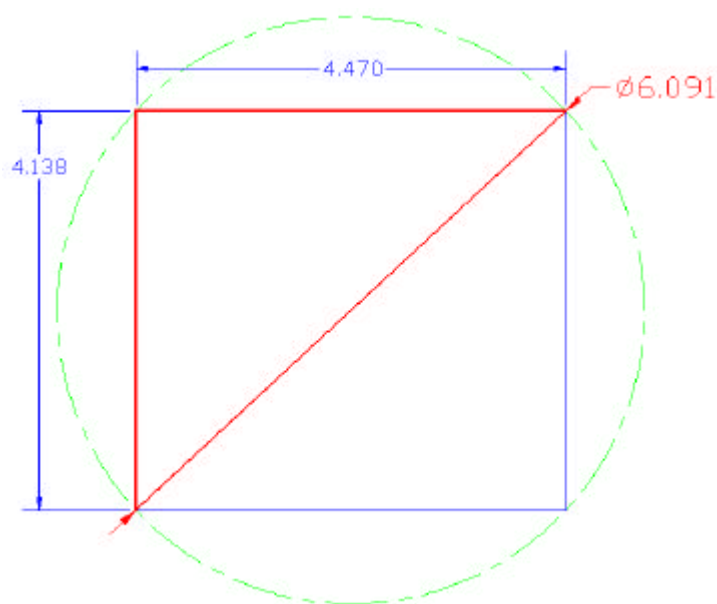
Referring to the tolerance block above we get:

$$(.390 + .010) + (3.545 + .005) + (.51 + .010) = 4.470$$

Do the same for the overall width (the vertical dimensions).



Now that we have the maximum possible overall dimensions we need to fit this box into a circle. The diameter of the circle will be the minimum size bar stock for this job. How do we fit it into a circle? By making it into a right triangle!



The hypotenuse of this triangle is the diameter of the circle.

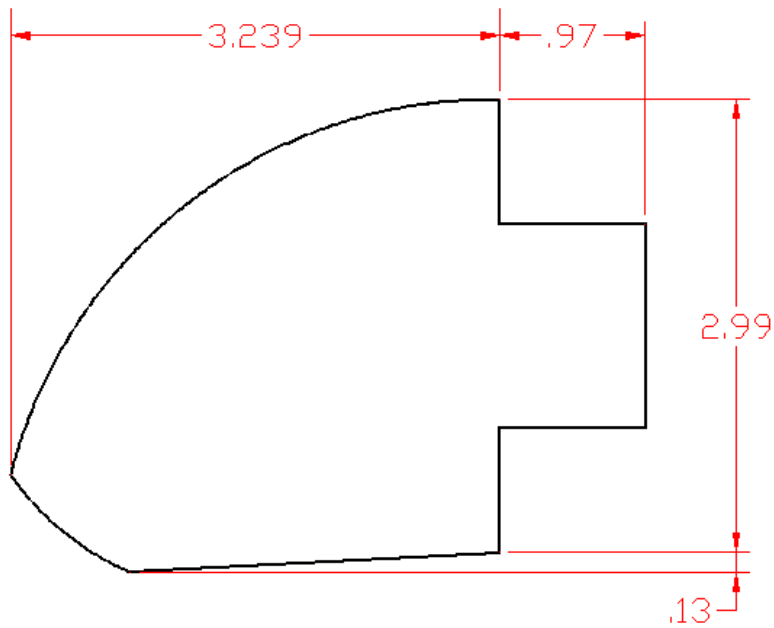
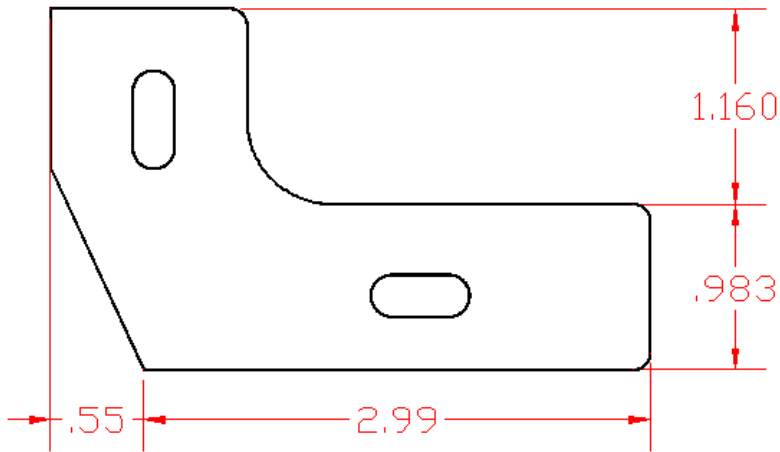
$$\sqrt{4.138^2 + 4.470^2} = 6.091$$

When the diameter is over 6", standard bar stock sizes usually come every 1/4". So for this part, the closest size above this minimum is probably 6.25" diameter.

Now use what you have learned to find the smallest bar stock you can to make blanks for the following parts. Remember to round your answer to the nearest 1/4".

Unless otherwise specified tolerances are:

- .XX ±.01
- .XXX ±.005
- .XXXX ±.0005
- Fractions ±1/64
- Angles ±1/2°



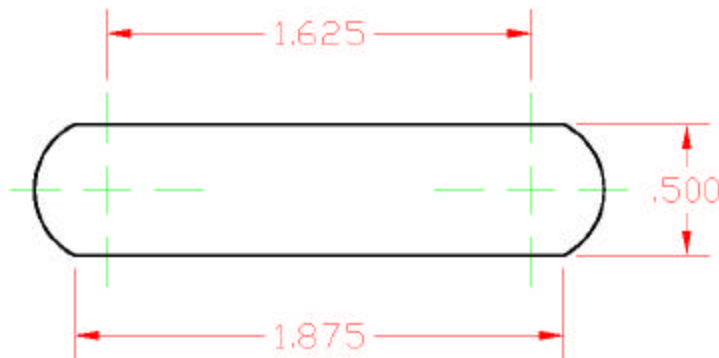
## Worksheet 8-5: Finding Missing Dimensions on Slotted Holes

While engineers design the overall product much of the design work and nearly all of the drafting is handed over to workers called “designers.” In manufacturing designers are often machinists and tool and die makers who moved up. They design piece parts and the tools that make these parts. While designers usually make complete drawings of the parts and tools that they design, in some companies they may pass the print on to an entry-level worker called a “detailer.”

The detailer’s job is to make sure that the drawing has all information, all the dimensions and notes, which the production departments will need to make the part. This is necessary because more specific information is needed to produce a part than to design it. Adding this extra manufacturing information takes time so engineers, designers and detailers try to include only the information that is absolutely necessary.

Sometimes dimensions that you need to make the part will not be on the print. This is OK as long as there is other information that you can use to calculate the missing dimensions. Of course it would save you time if the detailers put these dimensions on the print, but if the detailer is behind schedule, they will certainly try to cut corners by drawing only what they absolutely must. Then, it is up to the operator or the machinist to fill in the blanks. If you have the math skills to calculate these dimensions, you become a much more valuable player!

Look at the drawing of a slotted hole below:

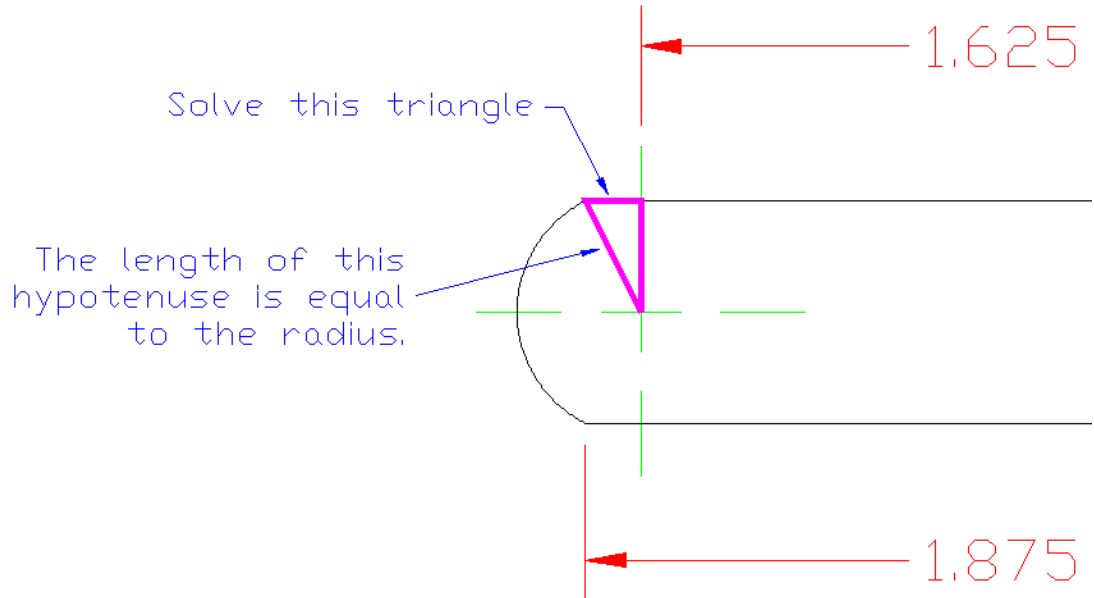


This could be a cutout for a display on a control panel. Note that each end of this slot has a radius that does not blend smoothly into the part.

Other slotted holes, like those on Worksheets 1-1 and 1-2, have end radii that blend smoothly. These slots are common on adjusting brackets, because a bolt diameter that matches the width of the slot will also match the end radius.

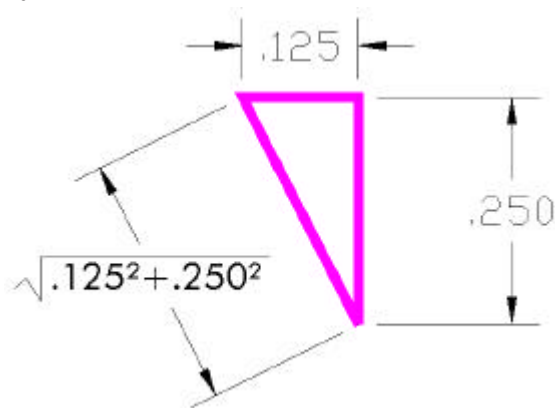
Display cutouts, on the other hand, don't have to match with bolts. They could have any radius on their ends. The problem here is that you need to know the radius in order to

make this cutout. Unfortunately, it is not given. How can you calculate it? With the Pythagorean theorem of course!



The print on the previous page gave the length of the flat edge of the slot as 1.875. It also gave the center-to-center distance between the end radii as 1.625. Finally, it gave the width of the slot as .500.

From this information we can draw a right triangle that will have a radius as its hypotenuse!



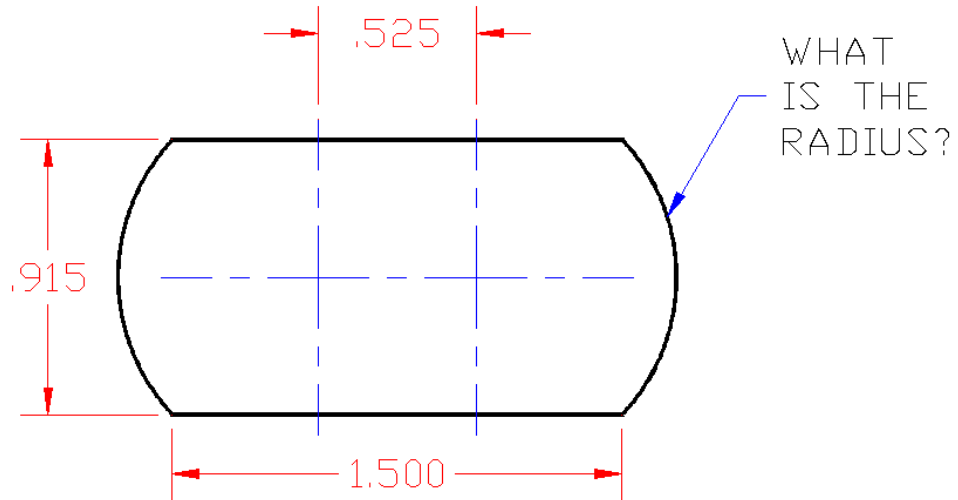
Can you see how this triangle was set up?

Calculate the length of the hypotenuse: \_\_\_\_\_

Now try to solve a similar problem on the next page.

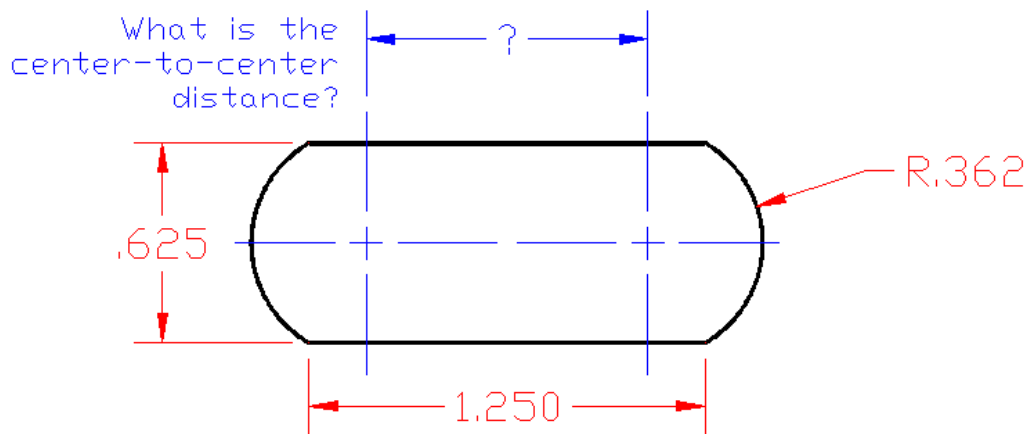


Problem 8-5-A: What is the end radius?



Problem 8-5-B: What is the center-to-center distance?

You still use the Pythagorean theorem, but now you have to solve for one of the sides of the triangle instead of the hypotenuse.



# Unit 9: Sines, Cosines, Tangents

## MEET IT

Trigonometric functions represent the relationships between an angle and the sides of a right triangle made from it. They can be very useful in determining the size of an angle when we know the dimensions of its sides. They can also help us project the image or “shadow” of an object onto a baseline or axis, when the object is at an angle to that axis.



Tom is a CNC machinist at an aerospace company. In the photo at right he is editing the program on a type of CNC lathe called a Swiss-style screw machine.

“You have to get a feel for what happens when you work with an object from different directions, for example when a tool comes in on an angle,” he says. “I use basic trigonometry to figure these things out. At first the trig functions like sine and cosine seemed weird, but soon I got the hang of it. Now I can calculate an angle from the linear dimensions and find the tangent point of a blend radius. That means I can find the start and the end of all the curves on the part, which you need to enter into the ‘G-code’ programs.”

## LEARN IT

In this unit you will learn to:

- Define sine, cosine, and tangent functions
- Remember the three main trigonometry formulas with “SOHCAHTOA”
- Determine unknown sides and angles of right triangles

## USE IT

- Solving triangles
- Depth of slots and chamfers
- Determining coordinates for bolt-hole circles
- Solving taper angles and CNC coordinates for an angled cut

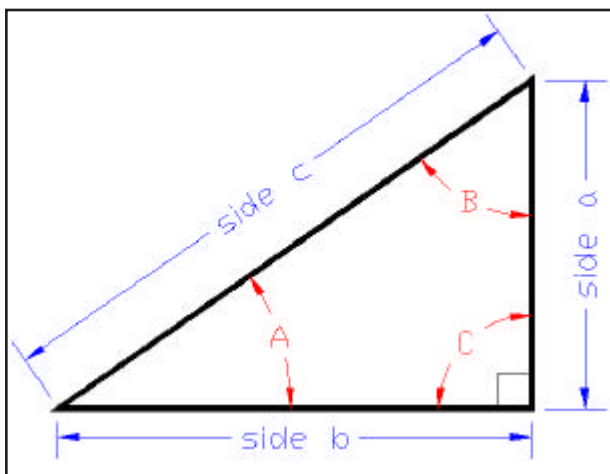
## LEARN IT

Trigonometry is the bread and butter of CNC programmers/machinists. It is also the critical math skill for many other trades including sheet metal layout and tool and die.

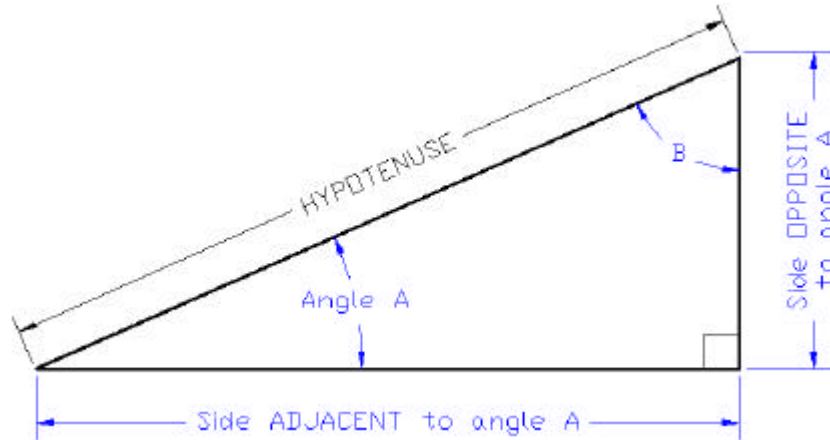
Trigonometry is triangle math. It is based on the ratios of the sides of right triangles. From these ratios come the three basic trig functions: sine, cosine and tangent.

If you draw a triangle and then photocopy it, you can use the copy machine to magnify or reduce the original and get a variety of triangles of different sizes. They will all have the same angles but their sides will have different lengths. These are called “similar triangles.”

Measure the sides of the original triangle and compare them to the photocopied triangles. You will find that the ratios of the sides are constant.



Use the right triangle on the left as an example. If you measure the length of side b (the bottom side) and divide it by the length of side c (the hypotenuse), you will find this same ratio in all the similar triangles that you can make on the copy machine. You will get the same number whenever you divide the bottom by the hypotenuse. Note that this works only for similar triangles so angles B and A are the same for all these triangles even though their lengths vary.



The diagram above shows a right triangle. From Unit 5 we know that the internal angles of a triangle =  $180^\circ$ . Since the lower right corner is a right angle ( $=90^\circ$ ), the other two angles must total  $90^\circ$ . Then if we know angle A, angle B will equal  $90^\circ$  minus A.

So, if we know one angle of a right triangle, we also know the other angle.

What's more, if we know the length of any two of the sides, we can figure out the angles. Finally, if we know only one angle and the length of only one side we can solve for the lengths of the other two sides and the other angle.

How? We can take advantage of the fact that the sides of similar triangles give the same ratio. This will give us a set of formulas that we can use to solve problems.

Look at the drawing above. We don't know the angles or the lengths of the sides but we can write formulas showing how they are related.

$$\text{Sine of angle A} = \frac{\text{length of the side OPPOSITE to angle A}}{\text{length of the HYPOTENUSE}}$$

$$\text{Cosine of angle A} = \frac{\text{length of the side ADJACENT to angle A}}{\text{length of the HYPOTENUSE}}$$

$$\text{Tangent of angle A} = \frac{\text{length of the side OPPOSITE to angle A}}{\text{length of side ADJACENT}}$$

These are the equations that define the three basic trig functions. The ratios have been calculated for every angle from  $0^\circ$  to  $90^\circ$  and are published in shop handbooks and trig tables. See the sample trig table below.

Angle in Degrees	sin	cos	tan
0	0.0000	1.0000	0.0000
5	0.0872	0.9962	0.0875
10	0.1736	0.9848	0.1763
15	0.2588	0.9659	0.2679
30	0.5000	0.8660	0.5774
45	0.7071	0.7071	1.0000
60	0.8660	0.5000	1.7321
75	0.9659	0.2588	3.7321
90	1.0000	0.0000	Infinity
53.13	0.8000	0.6000	1.3333
36.87	0.6000	0.8000	0.7500

The table at left shows the values for sine, cosine and tangent for a few common angles. Note that when you work with trig functions you always go one more decimal place with the function than with the dimension you are calculating. So this table would be useful for calculating dimensions with 3-place decimals but not four.

Better than the trig tables, a scientific calculator will give you the values for these trig functions quickly. They are labeled on your calculator as SIN, COS, and TAN. For example, enter 50 and hit the SIN key and you will get .7660.

If you get weird numbers like -0.2624 for the sine of 50 degrees, then your calculator is set for "radians" or "grads" instead of degrees. See your calculator manual to set to it to degrees. On the TI-30x the top of the display will say "DEG" in tiny letters when you have it right.

In manufacturing we only use degrees to measure angles. In engineering, math and science, we use radians for calculations and degrees for measuring.

Since trig values are known for all the angles, all we need to know is the length of one side and we can use our formulas to figure out the rest of the triangle! Of course solving trig formulas can be complicated so we will move ahead one step at a time.

**Example A:** Finding angles when you know the length of the sides. If the side opposite to angle A in right triangle is 2.165" long and the hypotenuse is 2.5", what are the angles?

**Step 1:** Decide which formula to use. You know the hypotenuse and the side opposite. Which trig function uses those? The sine function.

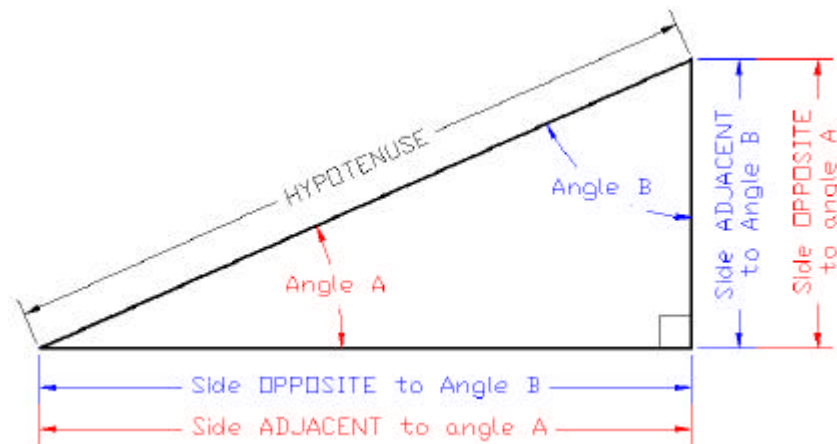
**Step 2:** Find the ratio:  $\text{sine of } A = 2.165/2.5 = .866$

**Step 3:** Match the trig value to the angle. Look up the value closest to .866 in the sine column of the table. Or you could enter .866 in your calculator and press the SIN key. What is the angle that has a sine of .8660? \_\_\_\_\_

Let's write a set of formulas for angle B as we did previously for angle A.

$$\sin B = \frac{\text{side Opposite to B}}{\text{HYPotenuse}} \quad \cos B = \frac{\text{side Adjacent to B}}{\text{HYP}} \quad \tan B = \frac{\text{side Opp. to B}}{\text{side Adj. to B}}$$

Now look at the triangle again, this time with all the labels for all the angles.



You can see that.

$$\sin B = \frac{\text{side Opposite to B}}{\text{HYP}} = \frac{\text{side Adjacent to A}}{\text{HYP}} = \cos A$$

Look at the trig table and you can see that  $\sin 60^\circ = \cos 30^\circ$

In a right triangle the sine of one angle will always be equal to the cosine of the other angle.

Example B: If you know the trig value, how can you find the angle that goes with it using your calculator?

Look for  $\text{SIN}^{-1}$   $\text{COS}^{-1}$   $\text{TAN}^{-1}$  on your calculator. These are called the “inverse” trig functions. In the TI-30x, these are the “2<sup>nd</sup>” functions assigned to the SIN, COS, and TAN keys. To find the angle that has a sine of .819 with a TI calculator, enter .819 in the display and hit the **2nd** key and then the **SIN** key. Your answer should be 55.

## SOHCAHTOA and the 9 Trig Formulas

To remember our three basic formulas, use the mnemonic, SOHCAHTOA. This stands for “Sine is Opposite over Hypotenuse Cosine is Adjacent over Hypotenuse Tangent is Opposite over Adjacent.” Remember that in the formulas below OPPOSITE and ADJACENT sides are relative to the particular angle. Only the HYPOTENUSE is constant.

$$\text{SIN} = \frac{\text{OPP}}{\text{HYP}} \quad \text{COS} = \frac{\text{ADJ}}{\text{HYP}} \quad \text{TAN} = \frac{\text{OPP}}{\text{ADJ}}$$

Rearranging these 3 basic formulas, we get 6 more:

$$\text{HYP} = \frac{\text{ADJ}}{\text{COS}} \quad \text{HYP} = \frac{\text{OPP}}{\text{SIN}}$$

$$\text{ADJ} = \text{COS} \times \text{HYP} \quad \text{ADJ} = \frac{\text{OPP}}{\text{TAN}}$$

$$\text{OPP} = \text{SIN} \times \text{HYP} \quad \text{OPP} = \text{TAN} \times \text{ADJ}$$

To these nine, add the two formulas from the Pythagorean theorem:

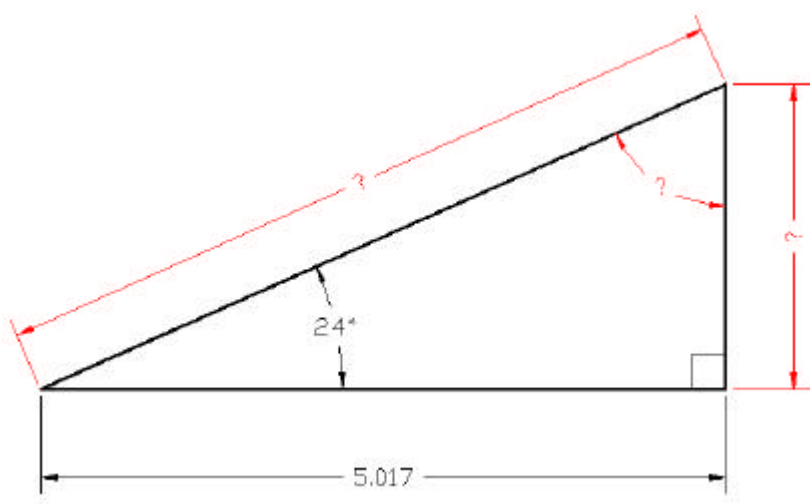
$$c^2 = \sqrt{a^2 + b^2} \quad a^2 = \sqrt{c^2 - b^2}$$

With this set of formulas we can now “solve triangles.” This is very powerful math. We can use this for many types of shop problems.

Caution: Please note that what we are doing in this unit only applies to right triangles. If a triangle does not have a 90° angle, we have to use more complicated equations and methods. However, don't give up if you don't see a right triangle. Any triangle can be cut into two smaller right triangles. Sometimes you can solve the right triangles inside the bigger triangle!

## Worksheet 9-1: Solving Triangles

Use the trig formulas and the Pythagorean theorem to find the missing sides and angles for the triangles below.



Example:

What do you know? You know an angle and you know the length of the side that is adjacent to it. This means you also know the other angle which is  $90 - 24 = 66^\circ$

What do you need to know? The opposite side and the hypotenuse.

Let's solve the hypotenuse first. What formula solves for the HYPotenuse by using ADJacent?

$$\text{HYP} = \frac{\text{ADJ}}{\text{COS}}$$

With a calculator or a complete trig table we get,  $\text{COS } 24^\circ = .9135$  (remember to round to at least 4 decimal places).

Then,  $\text{HYPotenuse} = 5.017 / .9135 = 5.492$

Now what formula can you use to get the OPPOSITE side?

$$\text{OPP} = \text{TAN} \times \text{ADJ}$$

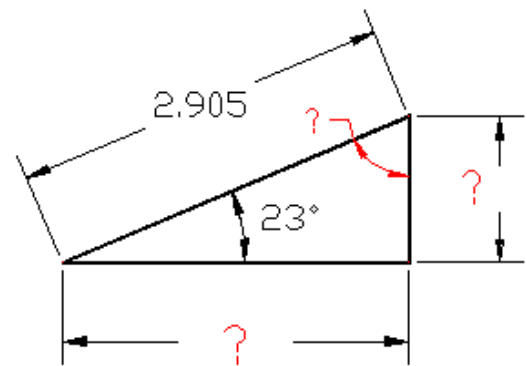
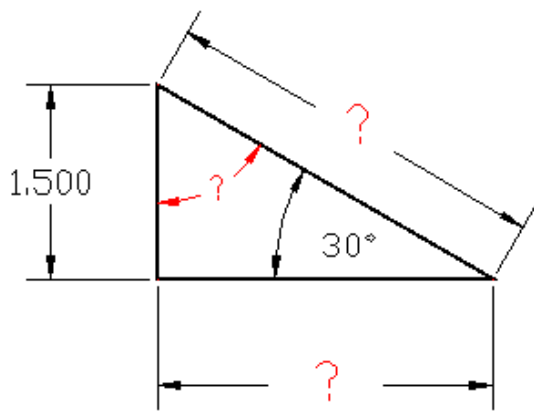
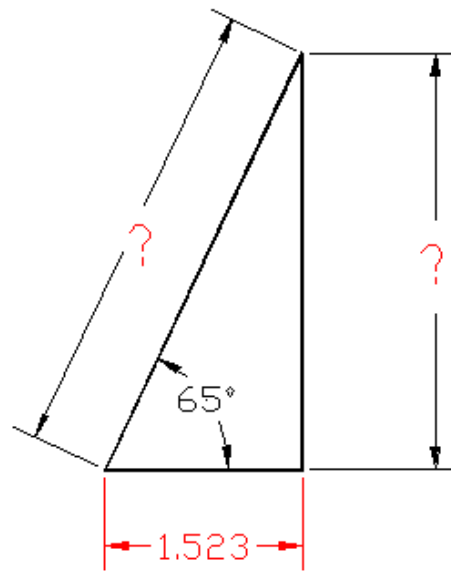
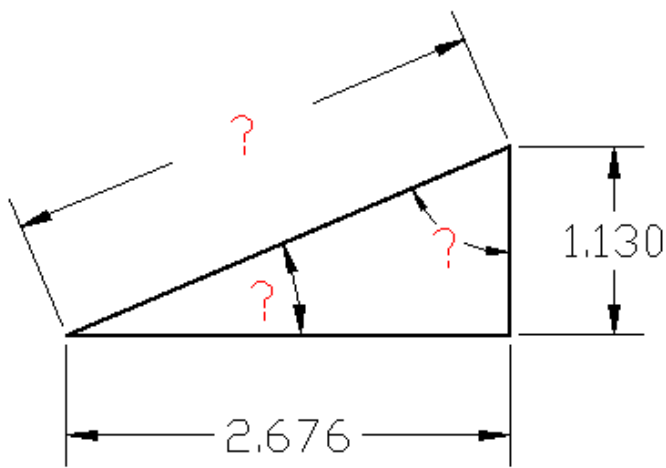


Keep in mind that TAN is the tangent of a specific angle and that ADJ means the side adjacent to this specific angle. We are looking for the side OPPOSITE to the 24° angle.

$$\text{Side OPPOSITE} = \text{Tan } 24^\circ \times 5.017 = .4452 \times 5.017 = 2.234$$

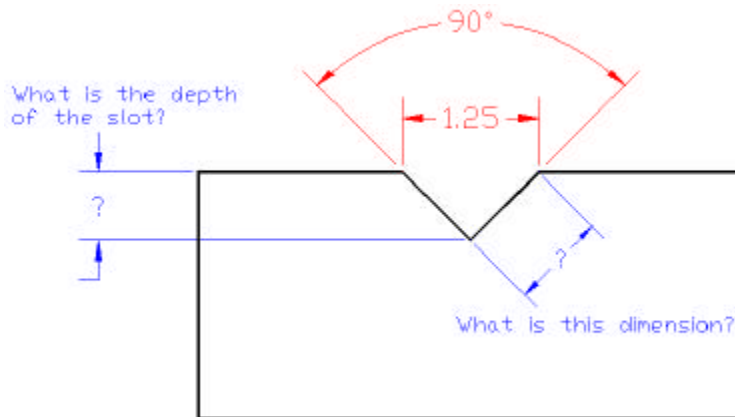
and we've solved this triangle!

Solve the four triangles below:



## Worksheet 9-2: Depth of Slots and Chamfers

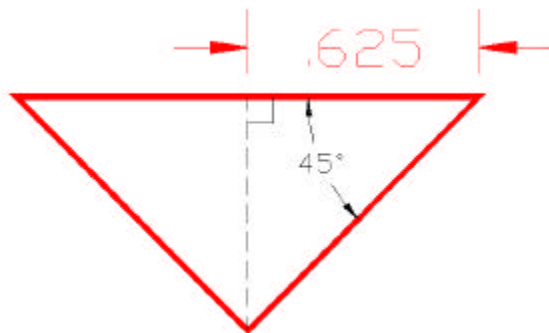
Look at the print below. The job calls for milling a slot with a V-shaped cutter with a 90° included angle. This means the sides of the cutter will be 45° from vertical. How deep do you have to go to make the opening 1.25”.



Look for the right triangle in the problem.

Write down what you know and what you need to know.

Although the V-slot has a right angle, it is not a right triangle that will give you the depth. So, cut the slot in half by drawing a line from the bottom of the V to the top surface of the part. This also cuts the 90 degree angle at the bottom of the slot in half. Note that this cut line forms a 90° angle at the top of the slot. See the drawing below.

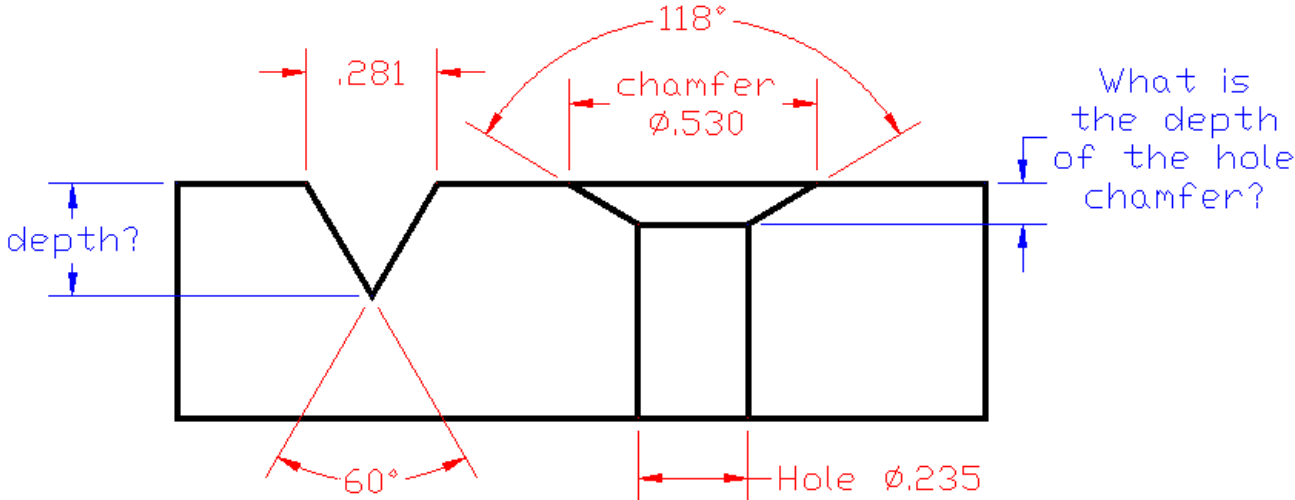


Now you have a 45° right triangle where the length of the side equals half the opening of the slot or,  $1.25/2 = .625$

Since this a 45° right triangle, the depth must also be .625. What about the other dimension? This is the hypotenuse of the 45° right triangle.

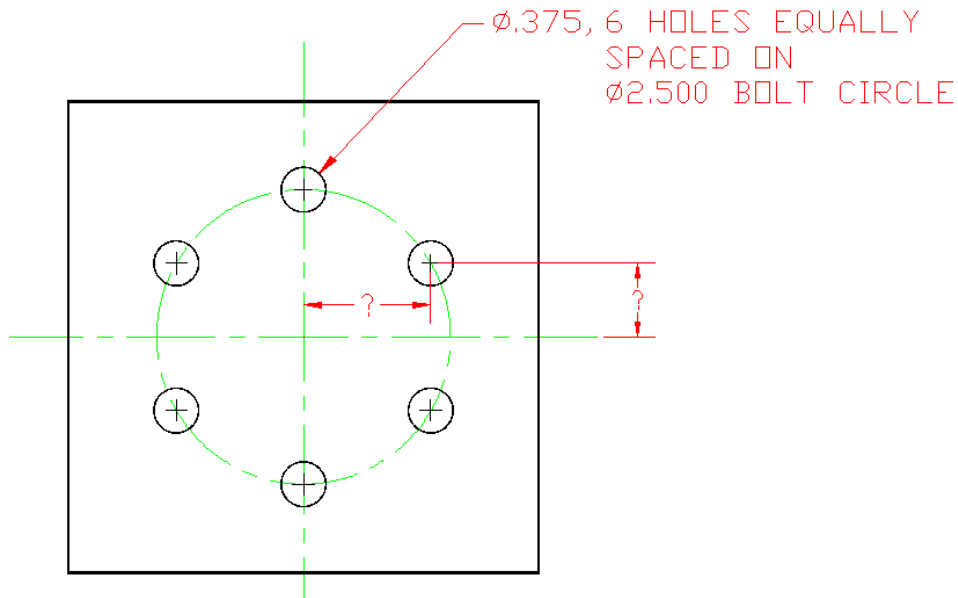
$$\text{HYP} = \text{ADJ} / \cos 45^\circ = .625 / .7071 = .884$$

Find the depth dimensions for the slot and the chamfer:

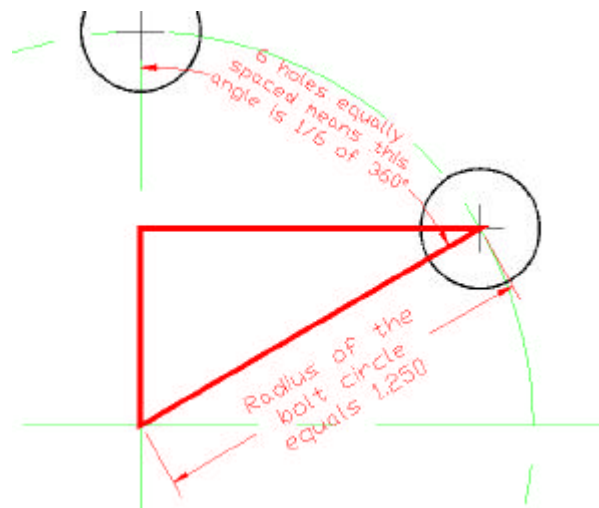


### Worksheet 9-3: Hole Locations on Bolt Circles

Look at the print below. Find the hole location for a hole on a "Bolt Circle".



To solve this problem you need to find the triangle inside the circle. Look at the sketch below:



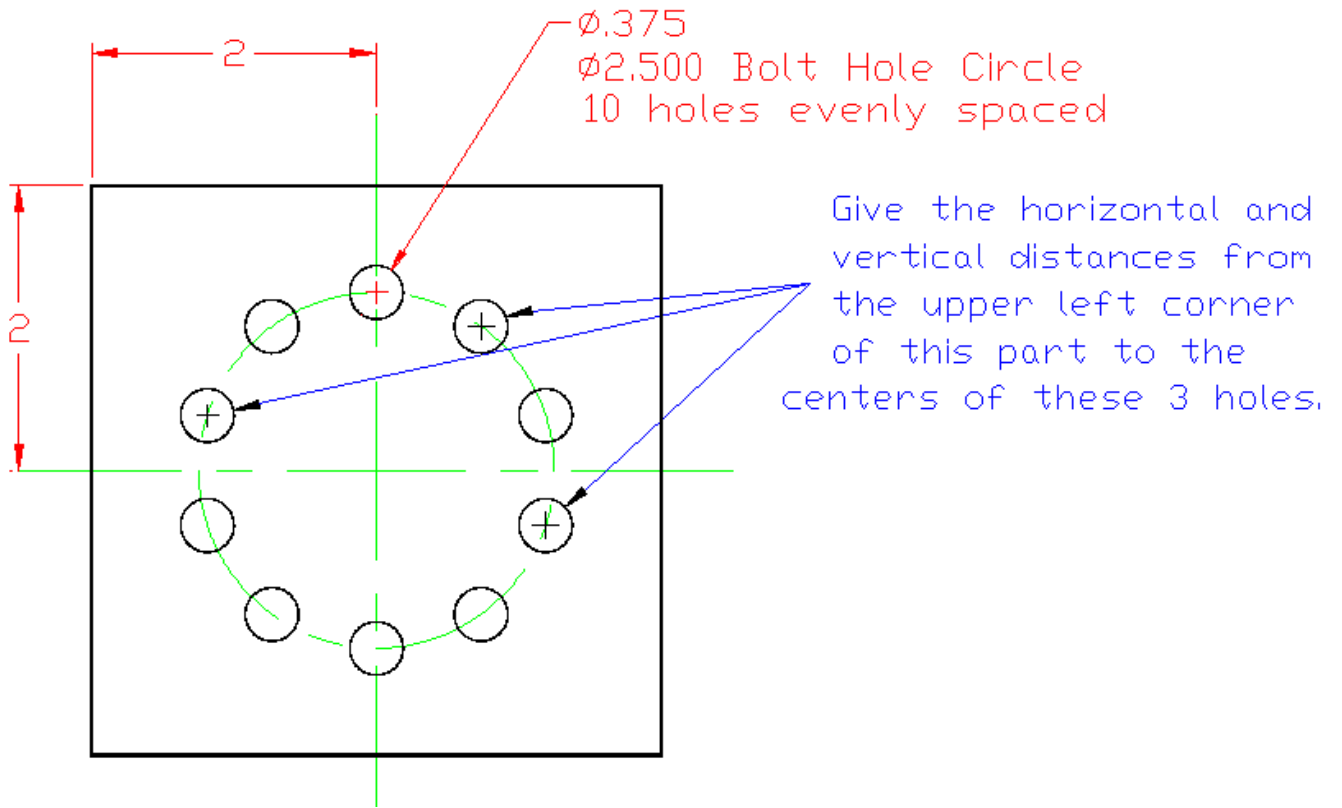
This triangle has a hypotenuse that is equal to  $\frac{1}{2}$  the diameter of the bolt circle:  
 $2.5/2 = 1.25$

We are not given an angle but we can calculate it from the hole spacing.  $1/6$  of  $360 = 60^\circ$

The side Opposite the  $60^\circ$  angle will give us the horizontal dimension. The side Adjacent will give us the vertical.

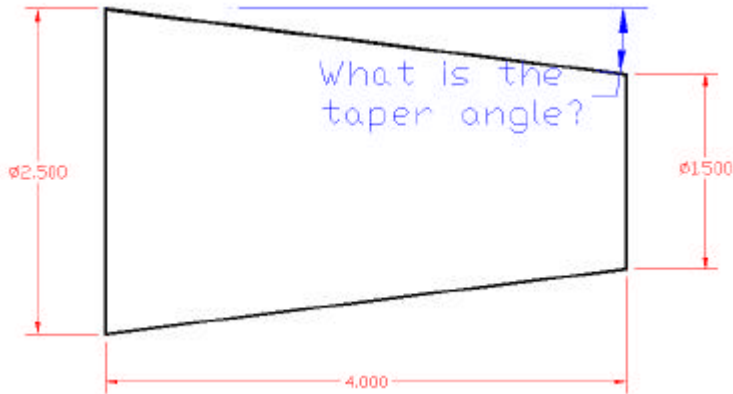
$$ADJ = \cos 60^\circ \times 1.25 = .5 \times 1.25 = .625 \text{ (vertical dimension)}$$

$$OPP = \sin 60^\circ \times 1.25 = .866 \times 1.25 = 1.083 \text{ (horizontal dimension)}$$

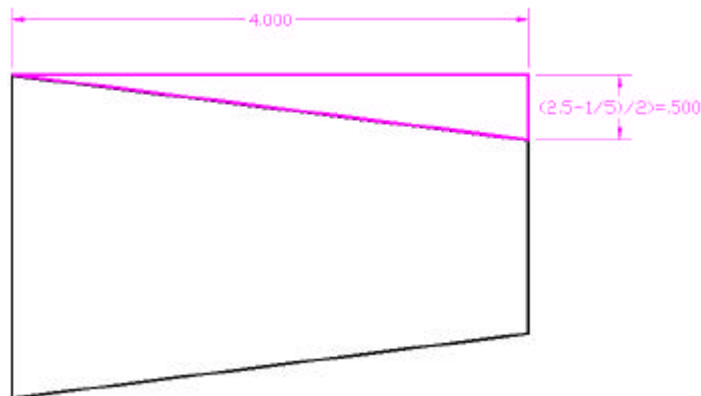


### Worksheet 9-4: Taper Angles

To cut a tapered shaft with a compound slide, you must know the taper angle. However, the angles are not always given on prints. Sometimes you have to calculate it.

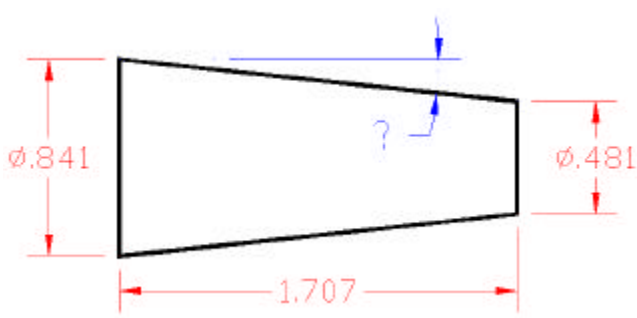


Can you see the triangle in this problem?

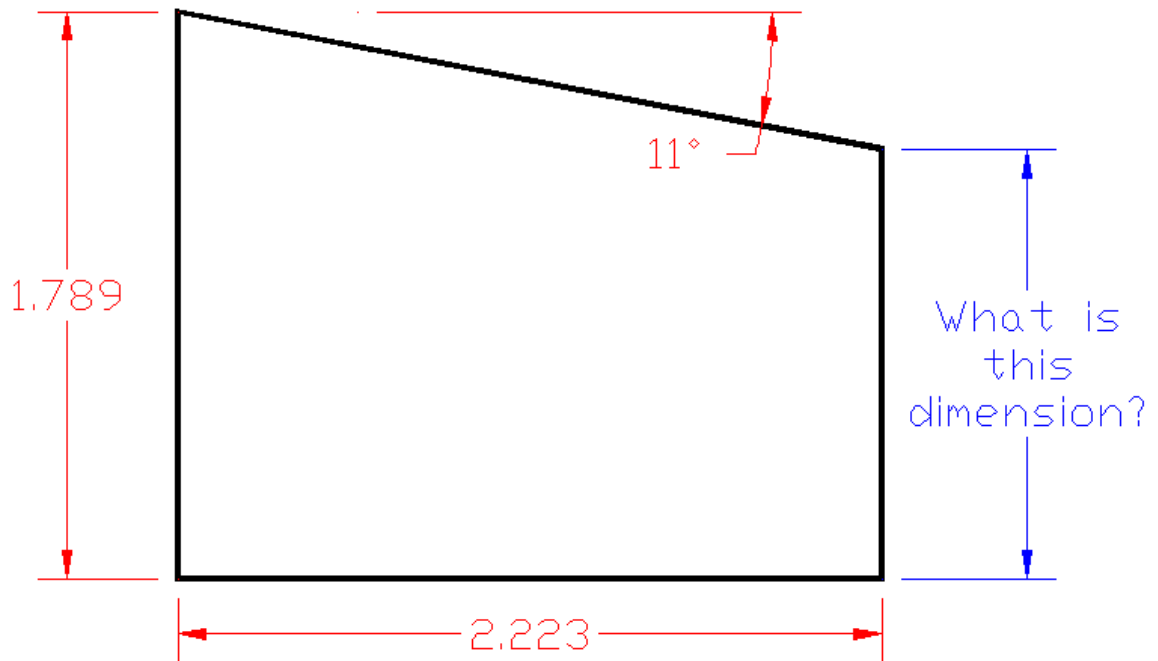


You know the side OPPOSITE to the taper angle and the side ADJACENT. What formula should you use? Calculate the taper angle and write your answer on the print above.

Use the print below to figure the taper angle:



A similar problem is finding the coordinates for angled cuts in CNC programming. Look at the sketch below:



Find the missing dimension.

(Hint: the print gives you the angle and one side of the triangle you have to solve.)

# Unit 10: Statistical Process Control

## MEET IT

Dolores sets up and operates a 100-ton OBI press at an automotive parts manufacturer. The parts made at the plant typically have tolerances of a few thousandths of an inch.

“Every manufacturing process has imperfections, and these really show up in precision work,” she says. “We have to carefully measure for SPC and we have to keep our eyes on the SPC charts to spot a problem before we start to make bad parts. When I spot a problem early, it can save the company thousands of dollars in scrap.”



## LEARN IT

In this unit you will learn to:

- Determine mean and median values
- Understand and define standard deviation
- Calculate standard deviation with a scientific calculator
- Fill out X-bar and R charts

## USE IT

- Centering the data: mean and median
- Measuring scatter in the data: standard deviation
- Calculating averages and ranges for SPC charts
- Filling out and interpreting an X-bar and R chart



## LEARN IT

Statistics is a branch of mathematics that is used to draw inferences about a population based upon information contained in a sample. By inferences we mean conclusions or even predictions about future events. We study a small sample, and we use the math of statistics to make conclusions about a much larger number of things. These things could be people who are going to vote or stamped parts coming off of a punch press.

During elections pollsters are paid to find out who people will vote for. They can't ask every voter. So they try to ask a random sample of voters, and using statistics, they try to predict how the election will turn out.

In manufacturing we use statistical process control (or SPC) to see if our process is in control. In effect, we are trying to predict the future: will our process continue to produce good parts, or will some parts be out of tolerance.

Just like the political pollsters, we cannot afford to check everything. Even if we could, we could never guarantee 100% good parts. Even the best human inspector will miss bad ones and reject good ones. In fact, QC Engineers often invoke the 80 -- 20 rule: human inspectors will usually miss 20% of the bad parts.

Yet, that's OK because we don't have to check every part (thankfully!). We only have to check the process. If the process is in control, the parts will be good, 100% good. We still have to check parts, but only a small sample, and we have to make calculations and plot our numbers onto charts. By keeping our eye on the chart, we can see if the process is drifting out of control and make a fix before we produce even one bad part!

What's more, computers now do most of the calculations and they also set off alarms when the charts indicate a problem. Still, the operators have to make accurate measurements and enter them correctly. It is also important for the operators to understand SPC charts, even if the computer has an alarm system.

## THE BASIC MATH OF STATISTICS

Statistics takes measurements of groups of numbers. There are really only two kinds of measurements that we make, although we have many different math tools in the statistician's toolbox:

- A) Where the center of the data is
- B) How spread out or dispersed the data is

The Average (or mean) is one measure of the center of the data. The median is another way to describe the center.

The standard deviation, on the other hand, will show us how spread apart the numbers are. The range is also a measure of how dispersed our data is.

### FINDING THE MEAN:

A basketball player's "average" is the number of points she or he scores in a typical game over a period of time.

Let's say, for example, that an NBA shooting guard plays 10 games in January and scores the following point totals in each game:

21, 23, 18, 19, 31, 15, 7, 22, 16, 20

The mean or average point production for this player would be:

$$\frac{21 + 23 + 18 + 19 + 31 + 15 + 7 + 22 + 16 + 20}{10}$$

= 19.2 points per game

We can write this with special math shorthand:

$$\text{Mean} = \bar{x} = \frac{\sum x}{n}$$

This says that the average of "x" (which we read as "x-bar") equals the sum total of all the x's (or the total number of points scored) divided by the number of x's (the number of games).

19.2 pts/game is a pretty good average, and we can see that this player is consistent. Still, we can get a better idea of the most likely score he will have in any particular game by looking at his median score.

The median is the middle number, and we can find it by writing out all the game scores in order from highest to lowest,

7, 15, 16, 18, 19, 20, 21, 22, 23, 31

Note that the 5<sup>th</sup> number is 19 and the 6<sup>th</sup> number is 20. The median is the middle number of the set so it will be between the 5<sup>th</sup> and 6<sup>th</sup> numbers. To get the median here, we have to find the number that is midway between 19 and 20.

The median then is  $(20+19)$  divided by 2 and that equals 19.5.

So the mean is 19.2 and the median is 19.5 points per game.

If the mean and the median are very close, then the numbers are centered around the average. So for every bad game, he also had a great game.

Other players are not so consistent. Some will score big against teams with a particular weakness (such as a center that is weak on defense) but score little against a strong front court.

Let's say that the basketball team's center scores the following points in a span of 9 games:

5, 6, 28, 33, 27, 7, 5, 4, 29

Calculate the mean:

$$\frac{\sum (5, 6, 19, 33, 27, 7, 5, 4, 29)}{9} = 15$$

Fifteen points per game is a good average. But let's find the median to see how consistent the center scores.

To find the median first line up the 9 point totals in order:

4, 5, 5, 6, 7, 19, 27, 29, 33

The 5<sup>th</sup> number in the set will be the middle number. It has 4 numbers on either side of it. So the median is: 7

While this player averages 15 points per game, his median score is 7. So this player is more likely to score 7 points than to score 15. His average gets a boost from the big scores against teams with poor defense in the paint.

## Worksheet 10-1: Mean vs. Median

Find both the mean and the median for sets of 10 measurements shown below.  
Remember that the mean is the average or:

$$\bar{x} = \frac{\sum x}{n}$$

and the median is the middle number.

Problem 1: Caliper measurement of O.A.L. (overall length)

The print dimension is  $1.275 \pm .010$ .  
Find the mean and the median. Circle any dimensions that are out of spec.

1.277, 1.267, 1.271, 1.265, 1.279, 1.284, 1.276, 1.286, 1.281, 1.282

The mean is: \_\_\_\_\_

The median is: \_\_\_\_\_

Problem 2: Micrometer measurement of O.D. (outer diameter)

The print dimension is  $.0778 \pm .0005$   
Find the mean and the median. Circle any dimensions that are out of spec.

.0774, .0778, .0780, .0773, .0779, .0781, .0782, .0781, .0784, .0782

The mean is: \_\_\_\_\_

The median is: \_\_\_\_\_

## Worksheet 10-2: Standard Deviation

Looking at the mean and the median will give you clues as to how spread out the numbers are, but to control a process, we need to get a really good measure of the variations from part to part. To do this we use a statistical calculation called a standard deviation. There are different ways to calculate standard deviation. In SPC we always use what's called the "n-1" standard deviation.

The standard deviation is represented by a small "s" or the Greek letter " $\sigma$ " (which is also a small "s" in the Greek alphabet).

To get the  $\sigma_{n-1}$  standard deviation we have to:

- 1) Subtract the average from each score.
- 2) Square each of these differences (that is, multiply the difference by itself).
- 3) Add up all the squared differences ("the sum of the squares")
- 4) Divide the squares by the number of games played (the number of data points)
- 5) Take the square root of that number.

This can all be summed up in one short but complicated equation:

$$\sigma_{n-1} = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

Don't worry if this is totally Greek to you. The calculator is going to do all the work.

Just to show how all the numbers come together, we will manually calculate a standard deviation. We will use the basketball players mentioned on page 105 as our example.

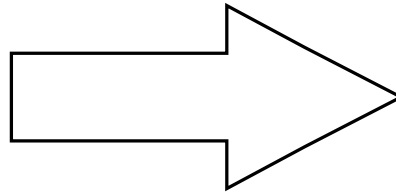
Over 10 games, the #2 guard averaged 19.2 pts/game. So,

$$\bar{x} = 19.2$$

Now we have to subtract the mean from each score:

- 21 - 19.2 = 1.8
- 23 - 19.2 = 3.8
- 18 - 19.2 = -1.2
- 19 - 19.2 = -.2
- 31 - 19.2 = 11.8
- 15 - 19.2 = -4.2
- 7 - 19.2 = -12.2
- 22 - 19.2 = 2.8
- 16 - 19.2 = -3.2
- 20 - 19.2 = .8

Square the answers:



- $1.8^2 = 3.24$
- $3.8^2 = 14.44$
- $-1.2^2 = 1.44$
- $.2^2 = .04$
- $11.8^2 = 39.2$
- $-4.2^2 = 17.64$
- $-12.2^2 = 148.84$
- $2.8^2 = 7.84$
- $-3.2^2 = 10.24$
- $+.8^2 = .64$

and then we add up the squared numbers: 343.56

So at this point we have:

$$\sum (x - \bar{x})^2 = 343.56$$

Since there were 10 games,  $n = 10$ , and  $n - 1 = 9$

so now we have:

$$\sigma_{n-1} = \sqrt{\frac{343.56}{9}} = \sqrt{38.1733} = 6.178$$

The standard deviation for this player's scoring is more than 6 points. This is a lot compared to the mean of 19.2, but by itself doesn't mean much in basketball. Good players will pass to their teammates when the defense covers them well. As long as someone else picks up the scoring when he has an off night, his team will be fine.

This is not so in manufacturing. We need a consistent process. We always want to see a small standard deviation, especially when compared to the total tolerance on the dimension that we are measuring. If we have to measure a slot width of  $.250 \pm .005$ , we don't want to see a standard deviation of .004. Our total tolerance is only .010; so we want to see a standard deviation that is a lot smaller than .010, something like .001.

For practice, let's calculate averages and standard deviations with a scientific calculator.

For the TI-30 calculator, we will use the following keys:

$\Sigma+$  The "sigma plus key," to enter data points type in the number than hit this key.  
The display will read : "n= 1"  
and each time you enter a point, the count will go up so your display will let you know how many data points you have entered.

After you hit all the points, type the keys labeled:

$\bar{x}$  the "X-bar key," this will give you the mean of the points you entered.

$\sigma_{n-1}$  Once the data points are entered, hit this key to get the "n-1 standard deviation" which is the standard deviation that we use for SPC.

## Exercise :

Use your calculator to find the average ( $\bar{X}$ ) and the standard deviation ( $\sigma_{n-1}$ ) for each of the following samples:

1.250	.063	.701
1.252	.061	.701
1.255	.057	.712
1.251	.058	.713
1.255	.060	.710

$$\bar{X} = \underline{\hspace{2cm}} \qquad \bar{X} = \underline{\hspace{2cm}} \qquad \bar{X} = \underline{\hspace{2cm}}$$

$$\sigma_{n-1} = \underline{\hspace{2cm}} \qquad \sigma_{n-1} = \underline{\hspace{2cm}} \qquad \sigma_{n-1} = \underline{\hspace{2cm}}$$

### Worksheet 10-3: Average and Range

This worksheet is similar to the last exercise. However, this time we will calculate the average ("X-bar") and the range (R) for each group of numbers.

The range is a very useful number. It turns out that often we can approximate the standard deviation from the range (R) or the average of ranges (  $\bar{R}$  , called "R-bar") .

You already know how to get the average. To get the range you subtract the smallest measurement from the highest measurement.

Example: In the exercise above the ranges would be calculated as follows:

1.250	.063	.701
1.252	.061	.701
1.255	.057	.712
1.251	.058	.713
1.255	.060	.710

1.255 biggest	.062 biggest	.713 biggest
-1.250 smallest	-.057 smallest	-.701 smallest

R = .005

R = .005

R = .012



Exercise:

For the groups of numbers below, calculate both the average and the range:

1.249  
1.250  
1.249  
1.249  
1.251

.063  
.069  
.065  
.060  
.066

.711  
.709  
.703  
.705  
.707

$\bar{x}$  = \_\_\_\_\_

$\bar{x}$  = \_\_\_\_\_

$\bar{x}$  = \_\_\_\_\_

R = \_\_\_\_\_

R = \_\_\_\_\_

R = \_\_\_\_\_

6.25  
6.29  
6.27  
6.31  
6.24

5.19  
5.25  
5.78  
4.99  
5.56

13.4  
13.4  
13.1  
13.8  
13.7

$\bar{x}$  = \_\_\_\_\_

$\bar{x}$  = \_\_\_\_\_

$\bar{x}$  = \_\_\_\_\_

R = \_\_\_\_\_

R = \_\_\_\_\_

R = \_\_\_\_\_

## Worksheet 10-4: Charting Data on an X-bar and R Chart

This page contains a table with 25 subgroups of 5 measurements.

1) Calculate the average and the range for each subgroup. Enter your answers in the table.

Spec limit	2.500 ±.010												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	2.497	2.503	2.503	2.492	2.497	2.506	2.500	2.502	2.500	2.501	2.502	2.493	2.496
2	2.499	2.502	2.500	2.498	2.491	2.501	2.502	2.499	2.504	2.493	2.490	2.488	2.502
3	2.502	2.509	2.505	2.494	2.497	2.492	2.509	2.504	2.503	2.495	2.495	2.509	2.507
4	2.502	2.499	2.504	2.497	2.495	2.505	2.501	2.497	2.507	2.500	2.498	2.492	2.501
5	2.493	2.500	2.505	2.501	2.502	2.499	2.502	2.511	2.498	2.503	2.498	2.499	2.509
<b>X-bar</b>													
<b>Range</b>													

	14	15	16	17	18	19	20	21	22	23	24	25
1	2.506	2.502	2.494	2.503	2.511	2.505	2.500	2.499	2.508	2.505	2.499	2.503
2	2.504	2.499	2.507	2.497	2.495	2.496	2.496	2.496	2.502	2.499	2.504	2.495
3	2.497	2.504	2.495	2.504	2.496	2.496	2.497	2.497	2.507	2.497	2.499	2.497
4	2.493	2.500	2.508	2.504	2.500	2.497	2.501	2.497	2.503	2.510	2.498	2.504
5	2.503	2.504	2.498	2.497	2.502	2.506	2.492	2.499	2.505	2.507	2.503	2.506
<b>X-bar</b>												
<b>Range</b>												

Note that 25 subgroups are typical on an SPC chart. Subgroups are often sampled at 1 hour intervals so a table like this would cover about 1 day's worth of production. Although many shops use computers to plot the SPC charts, it is still done manually in many instances, even in large plants.

2) Now calculate the average of the averages, or X-double-bar and the average of the ranges or R-bar:

$$\bar{\bar{X}} = \underline{\hspace{2cm}}$$

$$\bar{R} = \underline{\hspace{2cm}}$$

3) Now you will make an X-bar and R chart. The X-doublebar will be the center of the X-bar chart.

To do this you will use an SPC form from the automotive industry. The chart on the next page is reprinted with permission from the SPC Manual (Daimler-Chrysler, Ford, General Motors Supplier Quality Requirements Task Force).

a) Plot the 25 X-bar points on the chart on the next page.

b) On the same page but in a separate area underneath the X-bar chart, plot the 25 Ranges.

**CONTROL CHART**

PLANT	DEPT.	OPERATION	DATE CONTROL LIMITS CALCULATED	ENGINEERING SPECIFICATION	PART NO.
MACHINE NO.	DATES	CHARACTERISTICS		SAMPLE FREQUENCY	PART NAME

<b>AVERAGES (X BAR CHART)</b>																																																																																
$\bar{X} = \text{Average } \bar{X} =$	$UCL = \bar{X} + A_2 \bar{R} =$	$LCL = \bar{X} - A_2 \bar{R} =$																																																																														
$\bar{R} = \text{Average } R =$	$UCL = D_4 \bar{R} =$	$LCL = D_3 \bar{R} = *$																																																																														
<b>RANGES (R CHART)</b>																																																																																
<table border="1" style="width:100%; border-collapse: collapse;"> <tr> <th>DATE</th> <th>TIME</th> <th colspan="5">SAMPLES</th> <th>SUM</th> </tr> <tr> <td>1</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>2</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>3</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>4</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>5</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table>			DATE	TIME	SAMPLES					SUM	1							2							3							4							5																																									
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<b>ACTION INSTRUCTIONS</b>	<b>ACTION</b>
1.	ANY POINT OUTSIDE OF THE CONTROL LIMITS ON SPECIAL CAUSES
2.	A RUN OF 7 POINTS ALL ABOVE OR ALL BELOW THE CENTRAL LINE
3.	A RUN OF 7 POINTS UP OR DOWN
4.	ANY OTHER OBVIOUSLY NON-RANDOM PATTERN
5.	

<b>SUBGROUP SIZE</b>	<b>A<sub>2</sub></b>	<b>D<sub>3</sub></b>	<b>D<sub>4</sub></b>
2	1.88	*	3.27
3	1.02	*	2.57
4	.73	*	2.28
5	.58	*	2.11
6	.48	*	2.00
7	.42	.08	1.92
8	.37	.14	1.86
9	.34	.18	1.82
10	.31	.22	1.78

**THE PROCESS MUST BE IN CONTROL BEFORE CAPABILITY CAN BE DETERMINED.**

\* For sample sizes of less than seven, there is no lower control limit for ranges.

4) Analyze the chart. Do you recognize any patterns? Do you see any trends that might be a problem?

5) Now add the control limits to the charts. You will need to use the formulas provided. Once you have calculated the control limits, you draw horizontal lines across the charts to mark these limits:  
You need to plot

$$\text{Xbar Upper Control Limit, or } UCL_X = \bar{\bar{X}} + A_2 \bar{R} = \bar{\bar{X}} + (.58) \bar{R}$$

$$\text{Xbar Lower Control Limit, or } LCL_X = \bar{\bar{X}} - A_2 \bar{R} = \bar{\bar{X}} - (.58) \bar{R}$$

$$\text{Range Upper Control Limit, or } UCL_R = D_4 \bar{R} = 2.11 \times \bar{R}$$

6) With the control limits drawn, let's study the chart again. Are there any points outside of the control limits?

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# Glossary

ANSI (American National Standards Institute).....	An association that serves as a clearinghouse for nationally coordinated voluntary standards for fields ranging from information technology to building construction. Standards are established for such areas as definitions, terminology, symbols, materials, performance characteristics, procedures, and testing methods.
Bi-lateral Tolerance.....	The allowed variance in a dimension from a larger value (+) to a smaller value (-) than the specified (nominal) dimension.
Cartesian Coordinates ....	Coordinates whose axes are linear and perpendicular, wherein the position of any point can be defined by a set of coordinates along the axes. Also called rectangular coordinates.
Chamfer.....	A beveled or sloping edge (usually of a hole), dimensioned by giving either the length of the offset and the angle or a pair of linear dimensions.
CNC (Computer numerical control).....	A system in which a computer is used to precisely position tools and/or the workpiece and to carry out the sequence of operations needed to produce a part.
Cosine .....	In a right triangle, the ratio of the side adjacent to a given angle to the hypotenuse.
Dimensioning.....	Explicit indication of dimensions on a drawing or print.
GD&T (Geometric Dimensioning and Tolerancing).....	A system of dimensioning that controls the shape or form of part features (instead of just the size). GD&T ensures better assembly of parts and often provides greater tolerance for locations of features than conventional tolerancing. However, GD&T is a more complicated system and requires more math knowledge.



Histogram.....	A frequency distribution graph using bars, each of whose width represents a value range and whose height represents the frequency of measurements falling into that range.
Mean.....	The average of a group of values.
Median.....	In statistics, the actual or interpolated middle value in a group of values.
Micrometer.....	A precision tool capable of measuring to 0.0001 inch (about 0.01mm).
Pythagorean theorem.....	The theorem that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.
Range.....	In SPC, the largest measurement in a set minus the smallest one.
Sine.....	In a right triangle, the ratio of the side opposite a given angle to the hypotenuse.
SOHCAHTOA.....	An acronym to help in remembering the basic trigonometric function definitions.
SPC (Statistical Process Control).....	Using statistics to monitor the manufacturing process over time. SPC aims to identify trends that could produce rejects before any bad parts are made.
Specification.....	A detailed description of requirements, dimensions, or materials.

- Standard Deviation..... A measure of how far a data point is from the mean, which can indicate how dispersed data points are. The (n-1) standard deviation is the type of calculation that is used in SPC.
- Tangent..... In a right triangle, the ratio of the side opposite a given angle to the side adjacent to the angle.
- Taper..... A uniform increase or decrease in the diameter of an object, resulting in a wedge or conical shape.
- Tolerance..... A permissible deviation from a basic dimension.
- Unilateral Tolerance..... The total tolerance is in one direction only, either (+) or (-).
- Vernier Caliper..... A precision measuring instrument used for both inside and outside measurements, that is accurate to 1/1000 inch (0.001 inch) and 1/50 mm (0.02 mm).
- “X-Bar-and-R” Chart..... Chart of production data for a particular dimension showing the average value of a set of measurements and the range of values of the measurements. Also called an “Average and Range” chart.